NOTES ON OPEN CHANNEL FLOW

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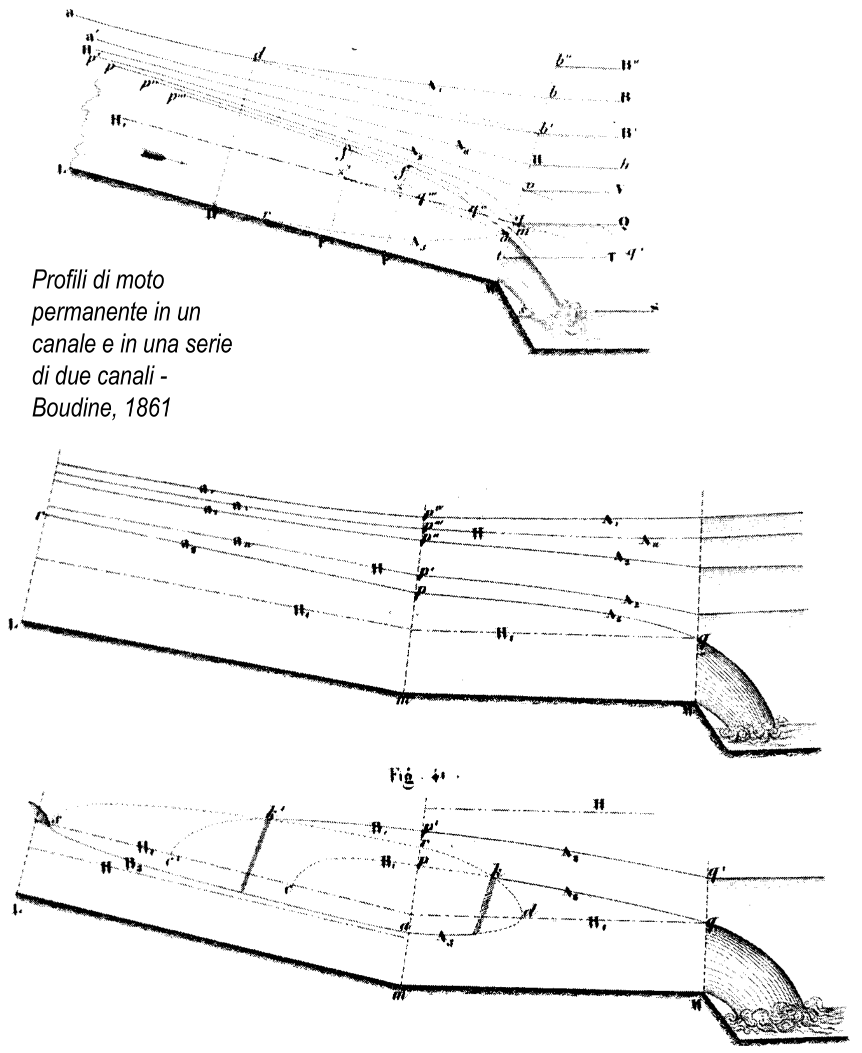


Profili di moto permanente in un canale e in una serie di due canali - Boudine, 1861

OPEN CHANNEL FLOW: glossary and main symbols used during the lectures

			Proprietà dei fluidi	**Fluid properties**
g	:	[m/s^2]	accelerazione di gravità	gravity acceleration
γ	:	[N/m^3]	peso specifico del fluido	Fluid specific weight
ρ	:	[Kg/m^3]	densità di massa del fluido	Fluid mass density
μ	:	[Ns/m^2]	primo coefficiente di viscosità dinamica	Viscosity coefficient
ν	:	[m^2/s]	coefficiente di viscosità cinematica	Kinematic viscosity coefficient
			Proprietà generali del moto	**General properties**
h	:	[m]	altezza della corrente	Water depth
R	:	[m]	raggio idraulico	Hydraulic radius
P	:	[m]	perimetro bagnato	Wetted perimeter
U	:	[m/s]	velocità media della corrente	Average flow velocity
Q	:	[m^3/s]	portata liquida	Liquid volumetric discharge
u	:	[m/s]	velocità puntuale della corrente	Local flow velocity
v_*	:	[m/s]	velocità d'attrito	Shear velocity
S_b	:	[-]	pendenza del fondo dell'alveo	Bed slope
S_f	:	[-]	cadente totale	Slope friction, Head slope
S_w	:	[-]	pendenza del pelo libero	Water surface slope
q	:	[m^2/s]	portata liquida in volume per unità di larghezza dell'alveo	Liquid volumetric discharge per unit length
q_i	:	[m^2/s]	portata in volume entrante per unità di lunghezza dell'alveo	As above, entering the flow
q_o	:	[m^2/s]	portata in volume uscente per unità di lunghezza dell'alveo	As above, going out of the flow
τ	:	[N/m^2]	sforzo superficiale	Shear stress
τ_0	:	[N/m^2]	sforzo medio al fondo	Average bed shear stress

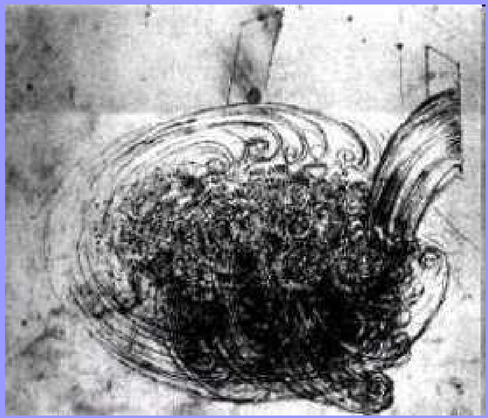
			Proprietà generali del moto	General properties
$z+p/\gamma$	:	[m]	carico piezometrico	Piezometric pressure
			Linea dei carichi totali	Total energy line
			Linea dei carichi piezometrici	Hydraulic grade line
S	:	[N]	Spinta totale della corrente	Specific Force
h_0	:	[m]	Profondità di moto uniforme	Normal depth
A	:	[mq]	Area bagnata della sezione retta	Wetted surface
B	:	[m]	Larghezza della sezione	Top width of the wetted surface
D	:	[m]	Profondità media della corrente = A(h)/B(h)	Hydraulic depth
$Q(h)$			Scala delle portate	Stage-discharge relationship

Termini generali	Words of common usage
bonifica	Land reclamation
fognatura	sewer
rigurgito	swellhead
Altezze coniugate	sequent depths or conjugate depths
Opera di presa/ griglia di presa	Intake / drop intake

	Numeri adimensionali	**Dimensionless groups**
Fr = $\frac{U}{\sqrt{gh}}$	Numero di Froude	Froude number
Re= $\frac{4UR}{\upsilon}$	numero di Reynolds della corrente	Reynolds number
X = $\frac{\rho v_* D}{\mu}$	numero di Reynolds del fondo	grain Reynolds number
Y = $\frac{\rho v_*^2}{\gamma_s D}$	numero di mobilità (sforzo adimensionale al fondo)	Dimensionless shear stress (mobility number)
Π_A	generica versione adimensionale di una propietà A del moto bifase	Dimensionless number corresponding to dimensional property A

OPEN CHANNEL FLOW: basic assumptions

Free surface flow: the upper surface is limited by a gas (tipically, the atmosphere) so that its pressure is constant
Tipical cases: channel (irrigation, hydropower supply, water supply, land reclamation…) river, sewer conduits, lakes…
Particular cases: groundwater flow, free surface flow in a syphon



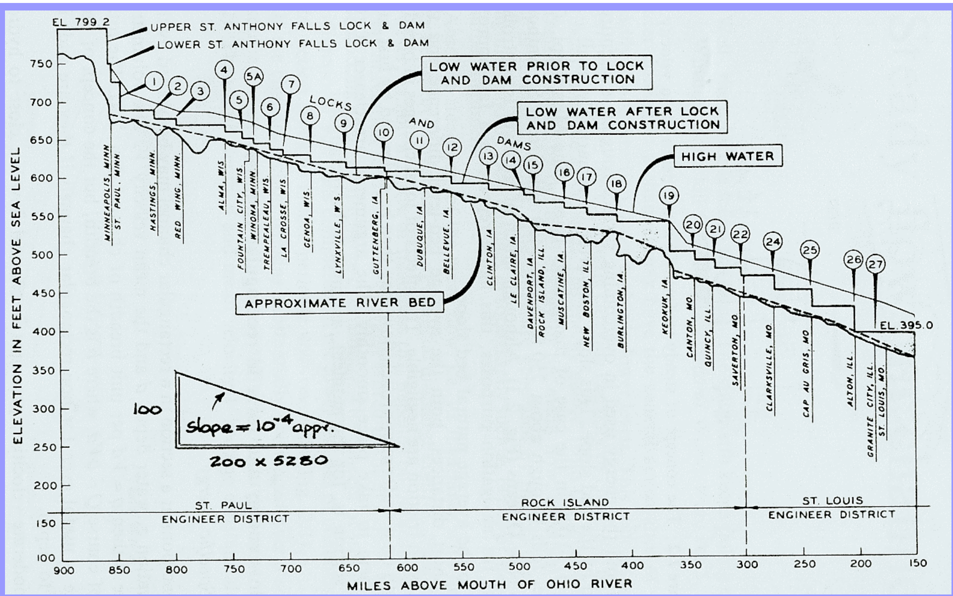
Main hypothesis in these lectures

- *1D approach in steady conditions*
- *Single-phase flow in unerodible, fixed bed*
- *Newtonian, constant density fluid*
- *Mostly, linear flow in rough turbulent conditions*

- *Bed slope $i_f < 0.1$ m/m*

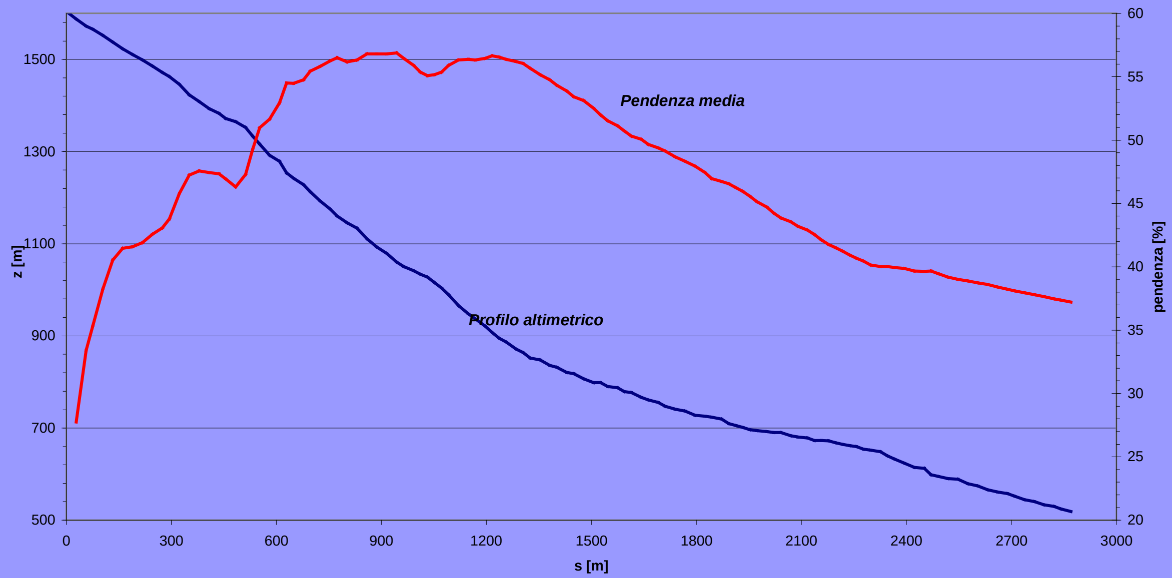
	Minimum slope	*Maximum slope*
Irrigation or land reclamation channel	0.0001	0.001
Sewer free surface pipe	0.001	0.05
Floodplain river	0.00005	0.005
creek	0.005	0.3

OPEN CHANNEL FLOW: *typical slopes*



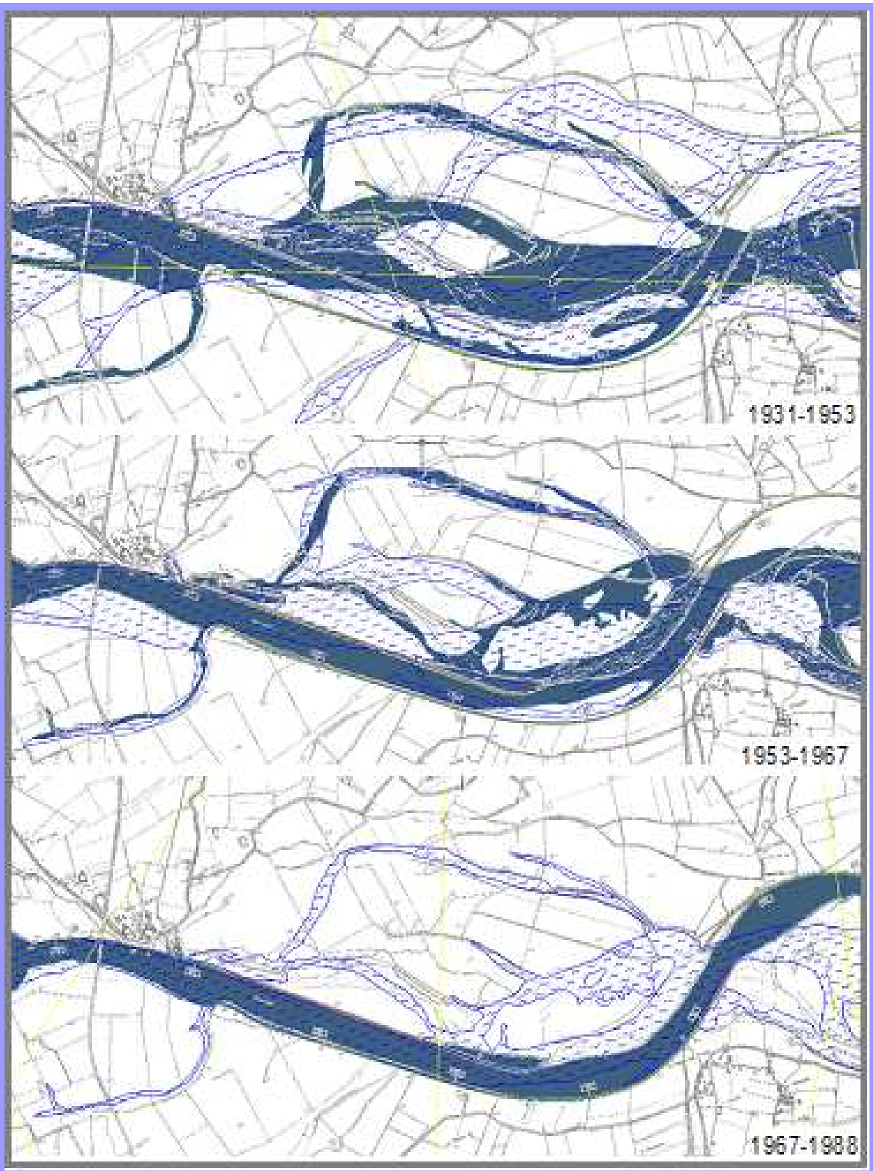
Mississippi river between St. Louis and Minneapolis (U.S. Corps of Engineers)

Typical mountain creek in Italian alps (T. Rossiga, A = 3.72 Km^2)



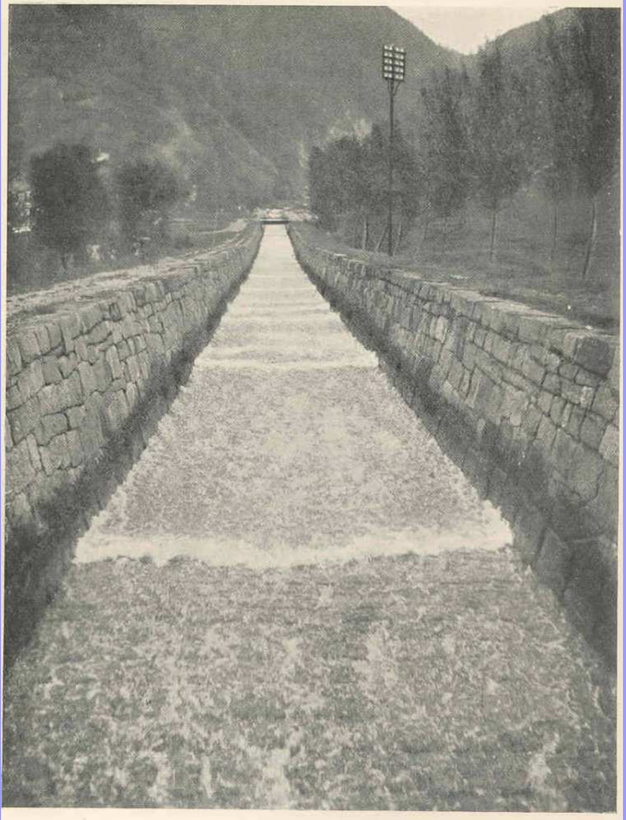
OPEN CHANNEL FLOW: relevance and applicability of basic hypothesis

Unerodible and fixed bed: the area surrounding Isola Pescaroli (from braided river to a single bed river)



OPEN CHANNEL FLOW: relevance and applicability of basic hypothesis

Roll waves in steep channels



ROLL-WAVES IN THE GRÜNNBACH CONDUIT, LOOKING UP-STREAM.

Debris flow (colata detritica) *(see USGS movie, by Costa and Garret)*

HOMEWORK:
see USGS movie on debris-flow

OPEN CHANNEL FLOW: mass balance

$$\int_A \rho(\vec{V}\cdot\vec{n})dA = Q_M$$

Mass discharge through A, the area of the cross section

$$\overline{\rho} = \frac{\int_S \rho dA}{A}$$

Average density on A

$$U = \frac{Q_M}{\overline{\rho}A}$$

Average velocity on A

$$\int_S \vec{V}\cdot\vec{n}dA = Q_V = Q$$

Volumetric discharge through A

$$\frac{D}{Dt}\left(\int_W \rho dW\right) = \frac{\partial}{\partial t}\left(\int_W \rho dW\right) - \int_A \rho(\vec{V}\cdot\vec{n})dA = 0$$

Mass balance for a control volume

$$\boxed{\frac{dW}{dt} = \int_A (\vec{V}\cdot\vec{n})dA = Q_{in} - Q_{out}}$$

(G.1) Mass balance for a basin, under constant density assumption

$$\frac{\partial(\overline{\rho}A)}{\partial t}dx = Q(x)_M - Q(x+dx)_M = \frac{\partial(\overline{\rho}UA)}{\partial x}dx$$

Mass balance for 1D flow

$$\frac{\partial(\overline{\rho}A)}{\partial t} + \frac{\partial(\overline{\rho}UA)}{\partial x} = 0$$

(G.1a) Mass balance for 1D flow when density varies (e.g.,turbiditic flow or debris flow)

$$\boxed{\frac{\partial(A)}{\partial t} + \frac{\partial(UA)}{\partial x} = q}$$

(G.1b) Mass balance for 1D flow when density is constant (typically, in open channel flow), and with net discharge q per unit length

HOMEWORK:
see exercises on Sarnico dam and on dam breach

$$\boxed{\frac{\partial H}{\partial s} = -\frac{\beta}{g}\frac{\partial U}{\partial t} - S_f}$$

(G.2) Energy balance equation for 1D gradually unsteady varied flow

$$H = z + \frac{p}{\gamma} + \frac{\alpha U^2}{2g} = z + h + \frac{\alpha U^2}{2g}$$

(G.2a) Total head

$$\alpha = \frac{\int_A u^3 dA}{U^3 A}; \qquad \beta = \frac{\int_A u^2 dA}{U^2 A}$$

First and second Coriolis' coefficient

$$\boxed{E = h + \frac{\alpha U^2}{2g} = h + \frac{\alpha Q^2}{2gA(h)^2}}$$

(G.2b) Specific Energy with respect to the thalweg

$$H = z + h + \frac{\alpha U^2}{2g} = z + E$$

Relationship between (G.2a) and (G.2b)

$$\frac{\partial H}{\partial s} = \frac{\partial z}{\partial s} + \frac{\partial E}{\partial s} = -S_b + \frac{\partial E}{\partial s}$$

$$\frac{\partial E}{\partial s} = S_b - S_f - \frac{\beta}{g}\frac{\partial U}{\partial t}$$

(G.2c) Energy balance in terms of E

$$\boxed{\frac{dE}{ds} = S_b - S_f}$$

(G.2d) Energy balance in terms of E in steady state conditions

OPEN CHANNEL FLOW: uniform motion

Let us consider a 1D flow in steady state condition with no lateral influx. Accordingly Q is constant

$$\boxed{\frac{\partial u}{\partial s} = 0}$$

(U.1) The definition of uniform motion in a weak sense, imply a cilindric boundary and can be immediately rewritten in term of average velocity

$$\frac{\partial U}{\partial s} = 0$$

That, if one consider (G.1b) , implies

$$\frac{\partial A}{\partial s} = 0$$

i.e, the wetted cross section does not vary. But in a prismatic boundary A=A(h), so that

$$\frac{\partial h}{\partial s} = 0$$

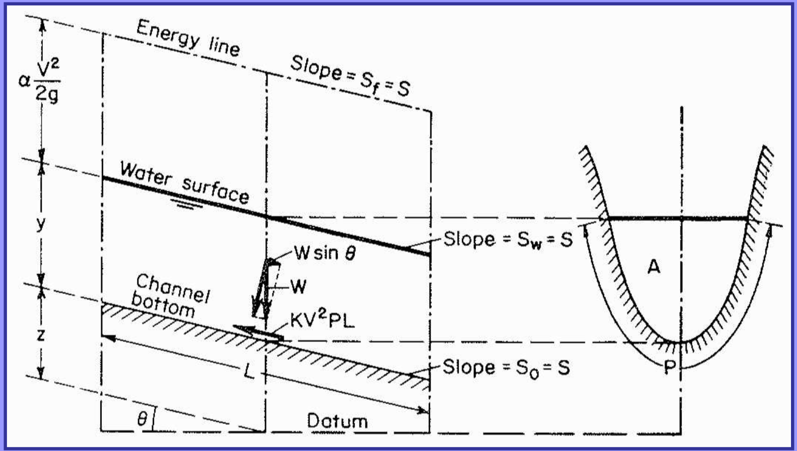
If now we consider (G.2b),

$$\frac{dE}{ds} = \frac{dh}{ds} - \alpha \frac{Q^2}{gA^3}\frac{dA}{ds} = 0$$

that implies

$$\boxed{S_b = S_f = S_w}$$

(U.2) Energy head loss = bed slope = water surface slope



The assumption (U.1) at the basis is never fully verified but it is often verified in an approximate way.
The implication (U.2) is of paramount importance because it implies a steady energy content of the flow
Accordingly, uniform flow is considered the reference state for all the other flow conditions

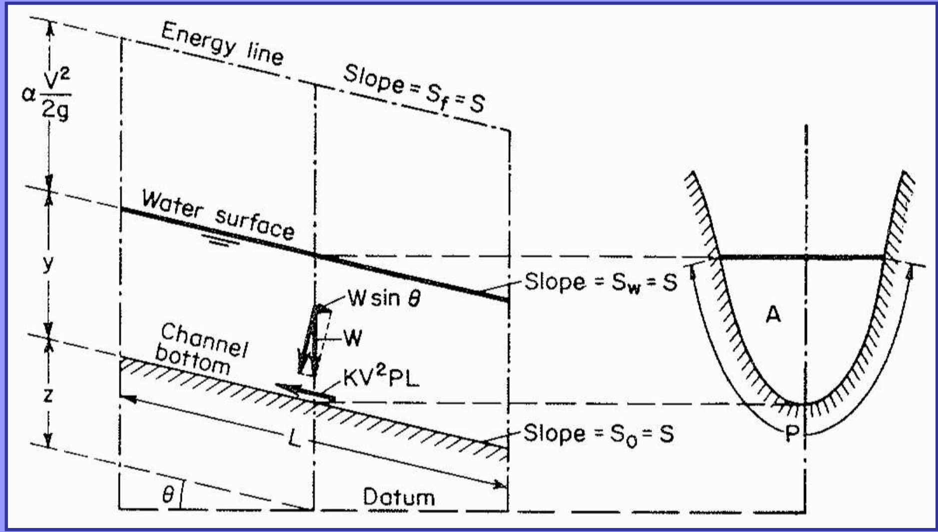
OPEN CHANNEL FLOW: uniform motion

Accordingly, from the kinematic point of view uniform flow is characterized by

$$\boxed{\frac{dU}{ds}=0; \quad \frac{dA}{ds}=0; \quad \frac{dh}{ds}=0}$$

Whilst, from the energetic point of view

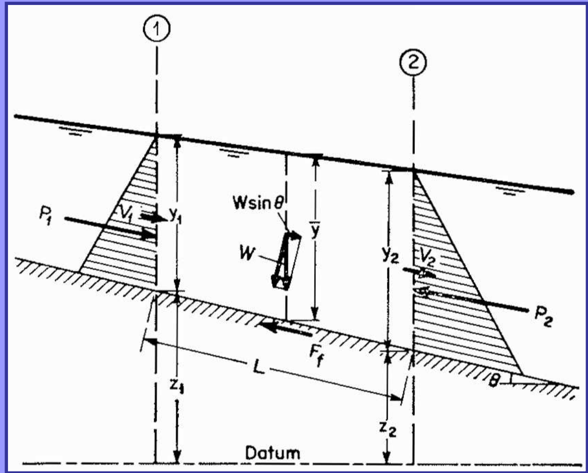
$$\boxed{\frac{dH}{ds}=-S_f; \quad S_b=S_f; \quad \frac{dE}{ds}=0}$$



And finally, from the momentum point of view

$$\beta\frac{\gamma Q^2}{gA_1}+\Pi_1+W\sin\theta=\beta\frac{\gamma Q^2}{gA_2}+\Pi_2+T_f$$

$$\boxed{W\sin\theta=T} \quad \Rightarrow \quad \gamma ALS_b=\tau_0 PL \quad \Rightarrow \quad \tau_0=\gamma RS_b=\gamma RS_f$$



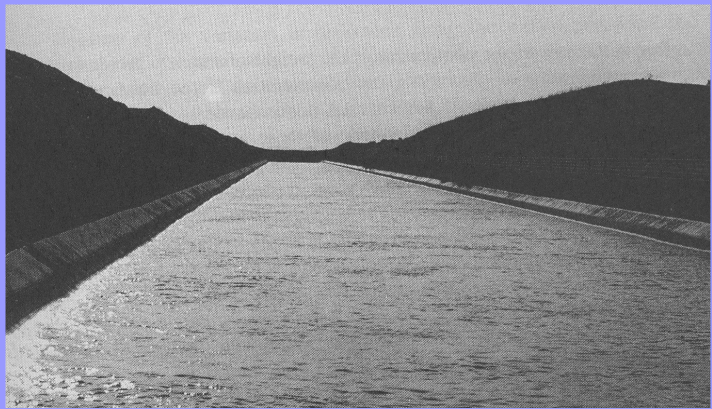
where

Π: pressure force acting on the given cross section;

W: weight of the water enclosed between the sections;

T_f: total external force of friction acting along the wetted boundary.

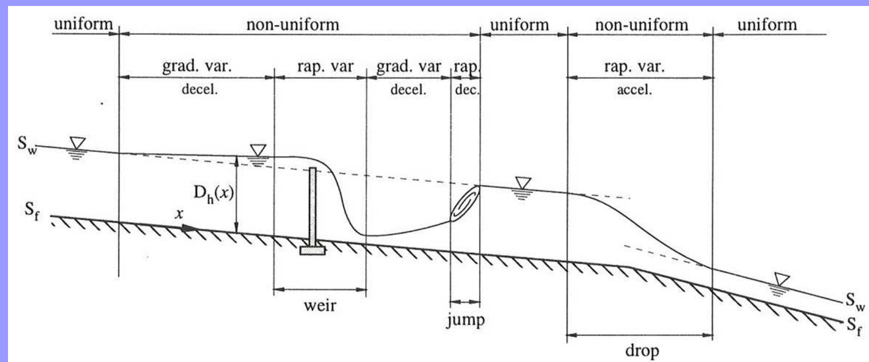
OPEN CHANNEL FLOW: uniform motion



In order to have a uniform flow, a prismatic channel is a necessary condition.

This channel, of trapezoidal cross section (b= 6m, B=17 m), is used to convey Q = 51 mc/s of drinkable water to a large american town. Its length is 300 kms.

However, this is not a sufficient condition because many man-made structures can interact with the flow causing departure from uniform flow (e.g., the gate on the left)
In these situations uniform motion still holds but one has to be sufficiently far away from the disturbance
How much far away is far ? We have to compute the profiles…



OPEN CHANNEL FLOW: uniform motion

Let us consider the problem of finding the relationship between h and Q in uniform flow

$$S_b = S_f = \lambda \frac{U^2}{8gR} = \lambda(\mathrm{Re}, \frac{\varepsilon}{R}, Fr, f) \frac{Q^2}{8gRA^2}$$

(U.3) Darcy-Weisbach relationship, with friction coefficient λ

$$\boxed{U = \chi\sqrt{RS_b} = k_s R^{1/6}\sqrt{RS_b} = \frac{1}{n} R^{1/6}\sqrt{RS_b}}$$

(U.3a) Chezy equation with Gauckler Strickler and Manning's coefficient

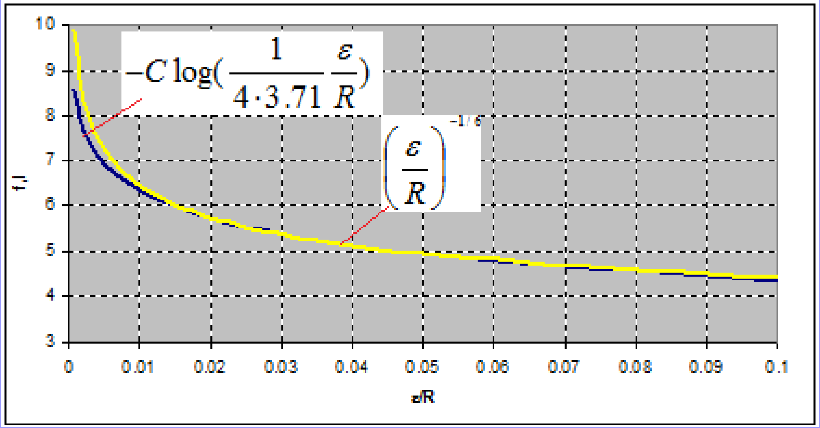
$$\boxed{Q = \chi A\sqrt{RS_b} = k_s R^{1/6} A\sqrt{RS_b} = \frac{1}{n} R^{1/6} A\sqrt{RS_b}}$$

(U.3b) Chezy equation with Gauckler Strickler and Manning's coefficient

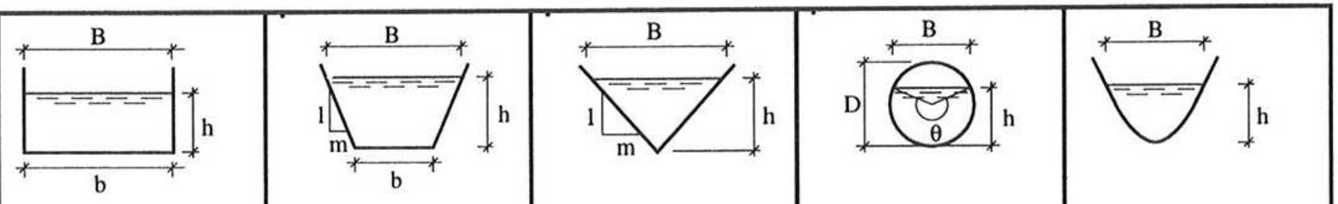
$$\chi = \sqrt{\frac{8g}{\lambda}}$$

By comparing (U.3) and (U.4a-U.4b) one sees that the friction coefficient and the Chezy coefficient have the same informative content. Actually, if one compare a logaritmic law for hydraulically rough flow for λ and admits that k_s is proportional to $\varepsilon^{-1/6}$

$$k_s R^{1/6} = \sqrt{\frac{8g}{\lambda}} \qquad \left(\frac{\varepsilon}{R}\right)^{-1/6} \approx -C\log\left(\frac{1}{4\cdot 3.71}\frac{\varepsilon}{R}\right)$$



Conclusion: law valid for hydraulically rough turbulent motion with k_s being a conveyance coefficient proportional to $\varepsilon^{-1/6}$



	Rectangle	Trapezoid	Triangle	Circle	Parabola
Section A	$b\,h$	$(b + mh)h$	mh^2	$\frac{1}{8}(\theta - \sin\theta)\,D^2$	$\frac{2}{3}B\,h$
Wetted perimeter P	$b + 2h$	$b + 2h\sqrt{1+m^2}$	$2h\sqrt{1+m^2}$	$\frac{1}{2}\theta D$	$B + \frac{8}{3}\frac{h^2}{B}$ *
Hydraulic radius R_h	$\frac{b\,h}{b + 2h}$	$\frac{(b + mh)\,h}{b + 2h\sqrt{1+m^2}}$	$\frac{mh}{2\sqrt{1+m^2}}$	$\frac{1}{4}\left[1 - \frac{\sin\theta}{\theta}\right] D$	$\frac{2B^2 h}{3B^2 + 8h^2}$ *
Width B	b	$b + 2mh$	$2mh$	$(\sin\theta/2)\,D$ or $2\sqrt{h\,(D-h)}$	$\frac{3}{2}\frac{A}{h}$
Hydraulic depth D_h	h	$\frac{(b + mh)\,h}{b + 2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta \angle \sin\theta}{\sin\theta/2}\right]\frac{D}{8}$	$\frac{2}{3}h$

* Valid for $0 < \xi \le 1$, with $\xi = 4h/B$. If $\xi > 1$: $P = (B/2)\left[\sqrt{1+\xi^2} + 1/\xi\,\ln\left(\xi + \sqrt{1+\xi^2}\right)\right]$

From W. H. Graf and M. S. Altinakar, 1998

OPEN CHANNEL FLOW: different formulation for friction - gravel bed rivers -

$$Q = \chi A \sqrt{R S_b}$$

$\chi = K_s R^{\frac{1}{6}} \quad K_s = 21 / D^{\frac{1}{6}}$ — *For natural channels with sediments of diameter D (D=D_{50} [m], Strickler, and D=D_{75}, Lane).*

$\chi = \frac{1}{n} R^{\frac{1}{6}}$ — *Manning*

$\chi = \frac{1}{n} R^{\delta} \quad \delta = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}(\sqrt{n} - 0.1)$ — *Pavloskii, 1925, took into account the exponent variation with relative roughness (0.1m <R< 3m ; 0.011<n <0.04)*

$\frac{\chi}{\sqrt{g}} = -5.75 \log\left[\frac{\chi}{\sqrt{g}\,\mathrm{Re}\, f} + \frac{\varepsilon}{13.3 R f}\right]$ — *Marchi (1961), for situations where a logarithmic profile holds. f is a shape factor varying between 0.8 (wide rectangular cross section) and 1.3 (triangular equilateral cs)*

$\frac{\chi}{\sqrt{g}} = 5.62 \log\left[\frac{aR}{3.5 D_{84}}\right]$ — *Hey (1979), for gravel bed rivers, where a varies with bed slope between 11.1 and 13.46.*

$\frac{\chi}{\sqrt{g}} = 5.62 \log\left[\frac{y}{D_{84}}\right] + 4$ — *Bathurst (1978) for rivers where slope is > 0.4%*

$n = 0.32 \frac{S_b^{0.38}}{R^{0.16}}$ — *Jarret (1992), for mountain creeks.*

$\frac{\chi}{\sqrt{g}} = \sqrt{\frac{8}{\lambda}} = 4.53 \log\left(\frac{R}{D_{50}}\right) + 3.09 \qquad \frac{\chi}{\sqrt{g}} = \sqrt{\frac{8}{\lambda}} = 5.41 \log\left(\frac{R}{D_{84}}\right) + 3.83$ — *Ferro e Giordano for gravel bed rivers*

$\frac{\chi}{\sqrt{g}} = 2.41\left[1 - 0.11\left(\frac{y}{D_{50}}\right)^{-1.1}\right] \ln\left[4.78\left(\frac{y}{D_{50}}\right)\right] \qquad \frac{\chi}{\sqrt{g}} = 2.41\left[1 - 0.45\left(\frac{y}{D_{50}}\right)^{-1.06}\right] \ln\left[2.73\left(\frac{y}{D_{50}}\right)\right]$ — *Butera e Sordo (1984), for beds with medium and high relative roughness*

OPEN CHANNEL FLOW: different formulation for friction

How do these formulas compare ? Let us consider an infinitely wide bed with y=1 m, $D=D_{50}$ = 0.2 m:

$$\chi = 21\frac{R^{1/6}}{D^{1/6}} = 27.5\ [m^{1/2}s^{-1}]$$ *Gauckler-Strickler:*

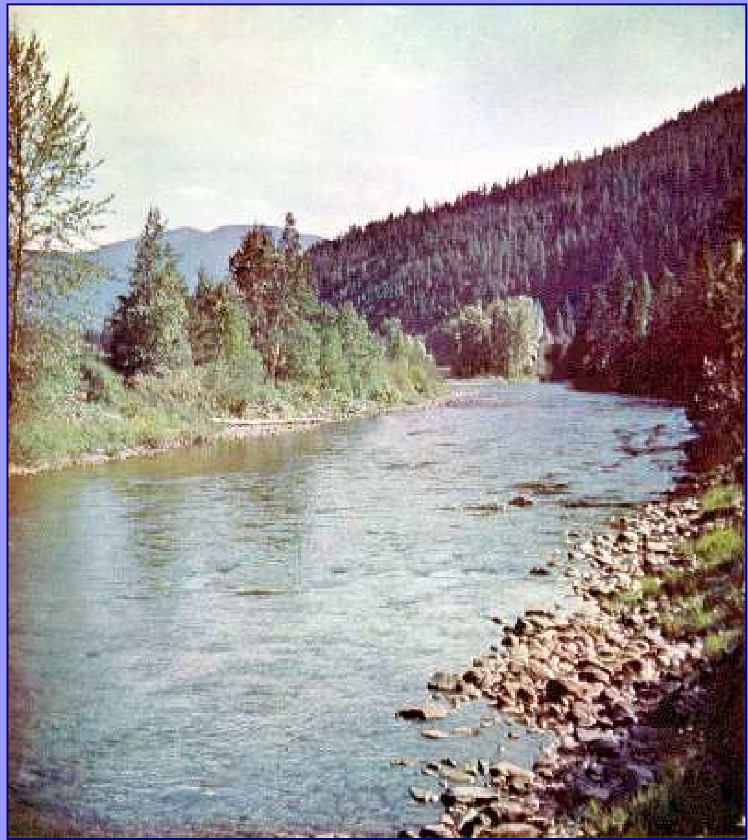
$$\chi = 19.6\ [m^{1/2}s^{-1}]$$ *Ferro e Giordano: first equation*

$$\chi_1 = 23.5\ [m^{1/2}s^{-1}]$$
$$\chi_2 = 18.1\ [m^{1/2}s^{-1}]$$ *Butera e Sordo: first and second equation*

$$\chi_1 = 21.7\ [m^{1/2}s^{-1}]$$ *Hey's equation with a = 12.*

Bathurst
$$\chi_1 = 24.8\ [m^{1/2}s^{-1}]$$

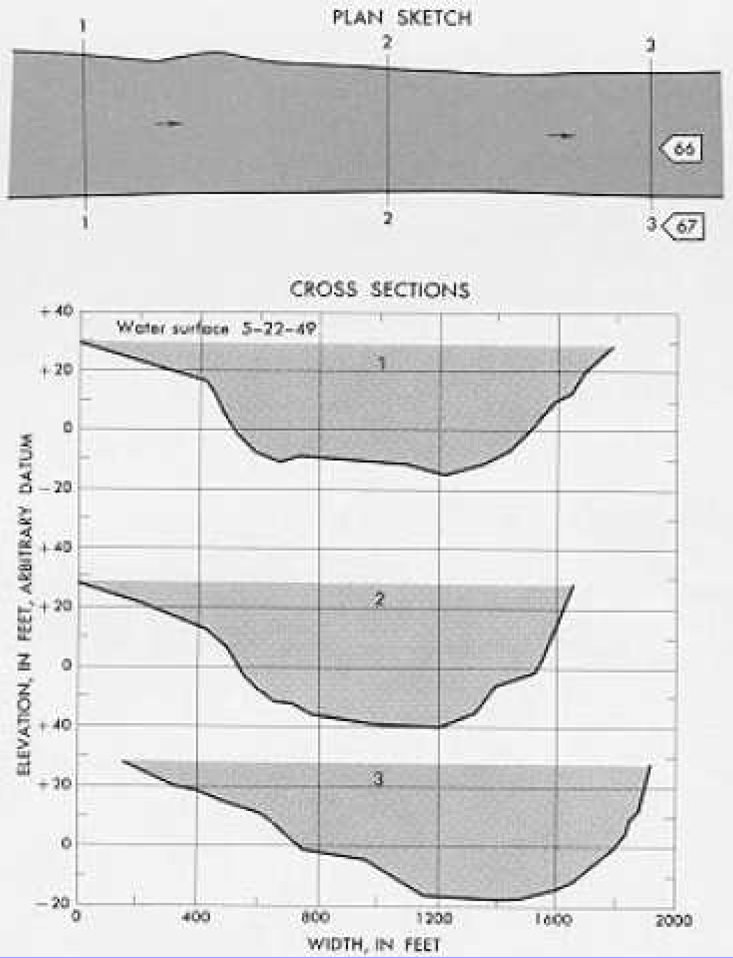
$$\chi_2 = 71.6\ [m^{1/2}s^{-1}]$$ *Marchi: with ε=D and we suppose hydraulically rough regime with f = 0.8*



Bed of gravel and well-rounded small boulders. Right bank is fairly steep and lined with trees and brush. Left bank slopes gently and has tree and brush cover.

OPEN CHANNEL FLOW: uniform motion

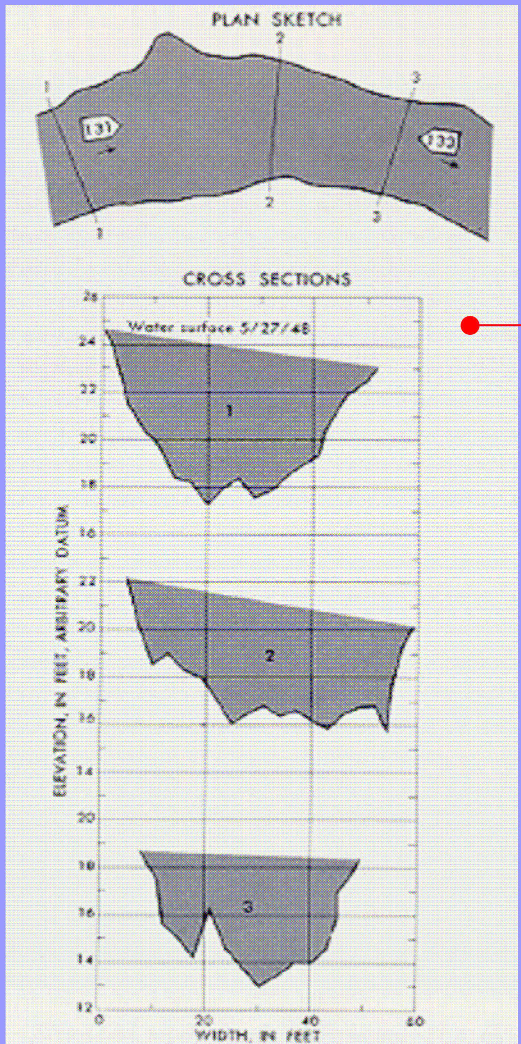
In river or streams the uniform flow is a mere approximation, still extremely usefull

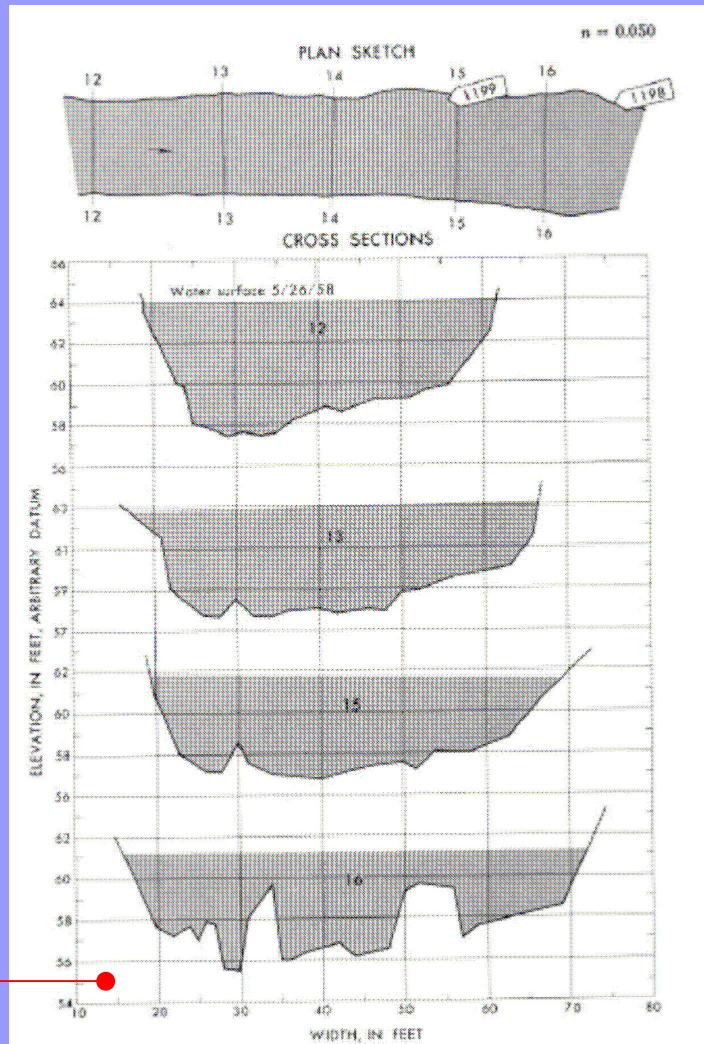


Sometime the approximation is very good, as in this case (plan sketch and cross sections, Columbia River at Vernita, Wash)

OPEN CHANNEL FLOW: uniform motion

In other cases it is a crude approximation, but still very usefull (plan sketch and cross sections of some creeks in the US, from Barnes, USGS)

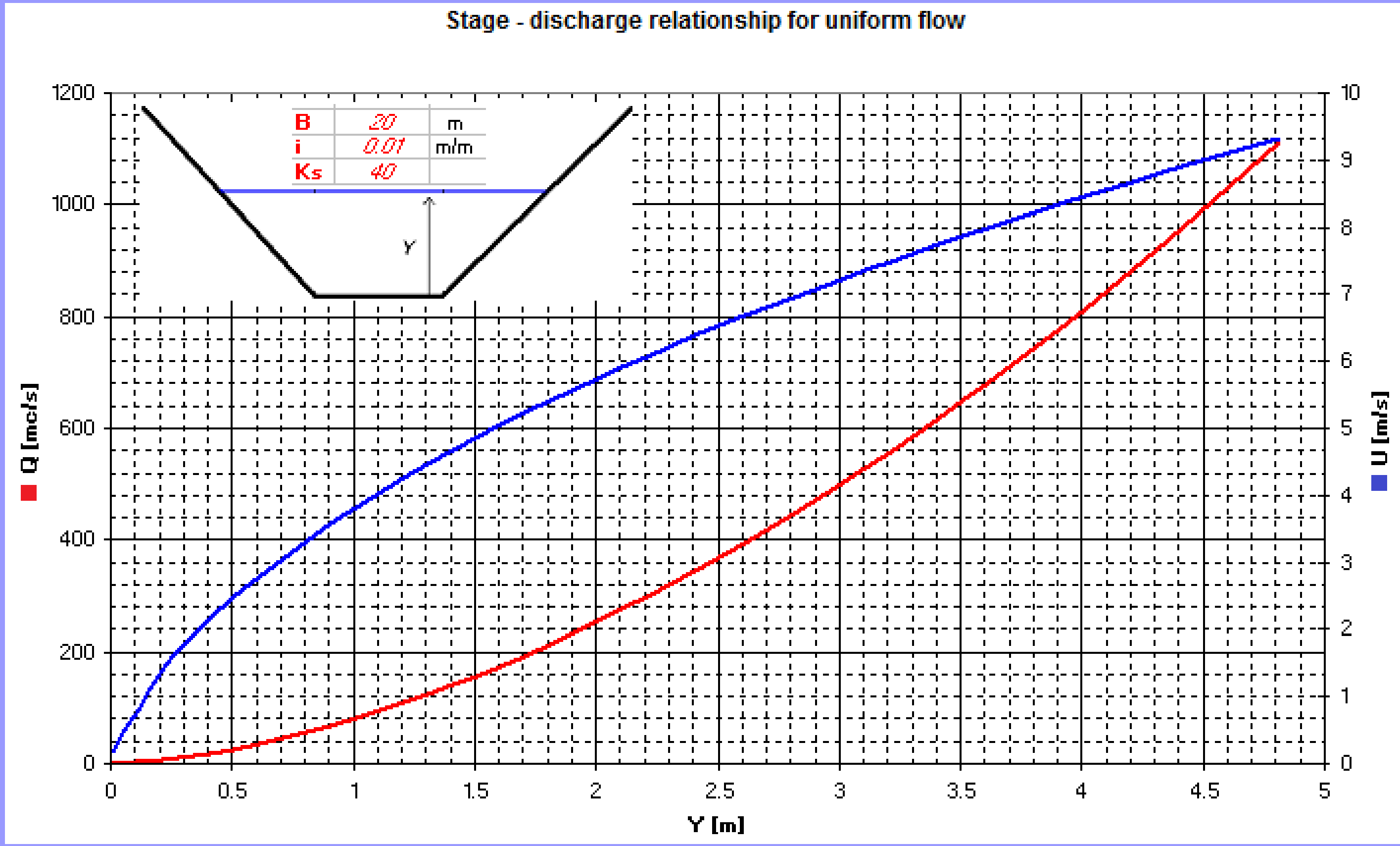




Stage-discharge (scala delle portate) relationship in uniform flow (also, normal rating curve)

$$Q = k_s R(h_0)^{1/6} A(h_0)\sqrt{R(h_0)S_b} = k_s \frac{A(h_0)^{5/3}}{P(h_0)^{2/3}}\sqrt{S_b}$$

The encircled expression is known as conveyance, being a function of h and representing a measure of capacity of water transport



$h = h_0$ is the so called NORMAL DEPTH (profondità di moto uniforme

OPEN CHANNEL FLOW: uniform motion

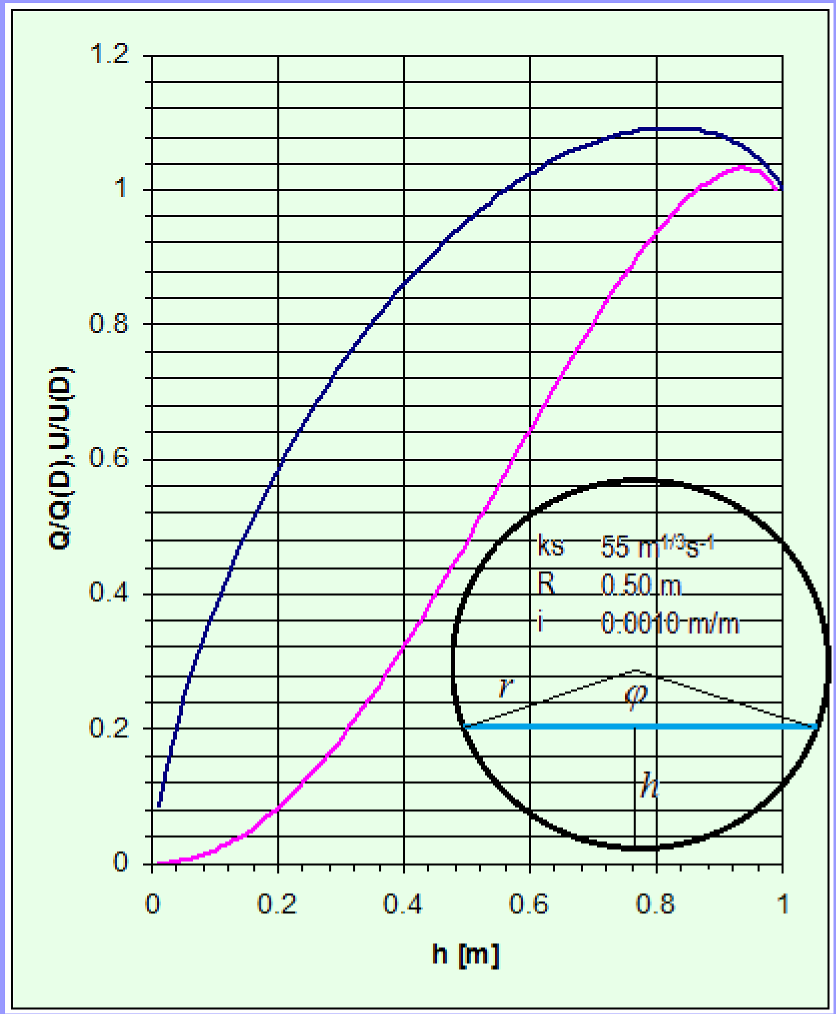
Stage-discharge relationship for closed conduits in uniform flow

$$\boxed{Q = k_s R(h)^{1/6} A(h)\sqrt{R(h)S_b} = k_s \frac{A(h)^{5/3}}{P(h)^{2/3}}\sqrt{S_b}}$$

$$A = \frac{1}{2} r^2 (\varphi - sen(\varphi))$$

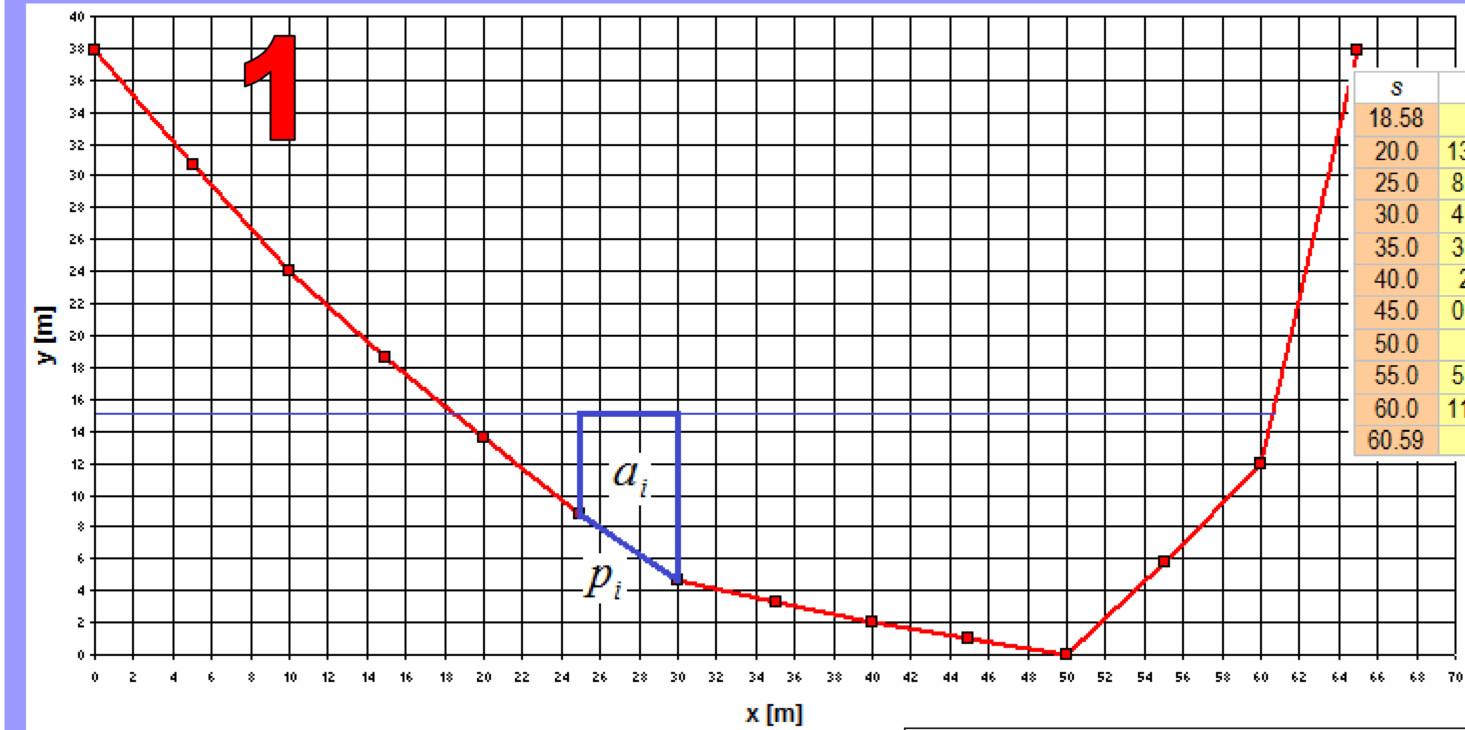
$$P = r\varphi$$

$$R = \frac{1}{2} r (1 - \frac{sen(\varphi)}{\varphi})$$



HOMEWORK:
Compute these relationship by using a spreadsheet

Stage-discharge relationship for irregular cross section in uniform flow



2

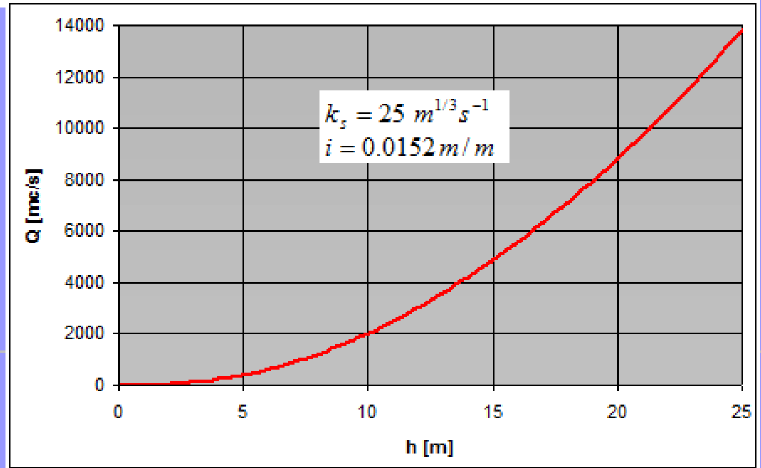
s	*z*	*p*	*P*	*a*	*A*	*R*
18.58	15	0.00	0.00	1.02	1.02	
20.0	13.564	2.02	2.02	19.17	20.19	
25.0	8.766	6.93	8.95	41.56	61.75	
30.0	4.609	6.50	15.45	55.35	117.1[illegible]	
35.0	3.251	5.18	20.63	61.80	178.[illegible]	
40.0	2.03	5.15	25.78	67.55	246.45	
45.0	0.947	5.12	30.89	72.64	319.09	
50.0	0	5.09	35.98	60.62	379.71	
55.0	5.753	7.62	43.60	30.83	410.54	
60.0	11.914	7.93	51.54	0.92	411.46	7.52
60.59	15	3.14	54.68			

$$P = \sum_{j=1}^{n} p_i \qquad A = \sum_{j=1}^{n} a_i$$

$$R = \frac{A}{P}$$

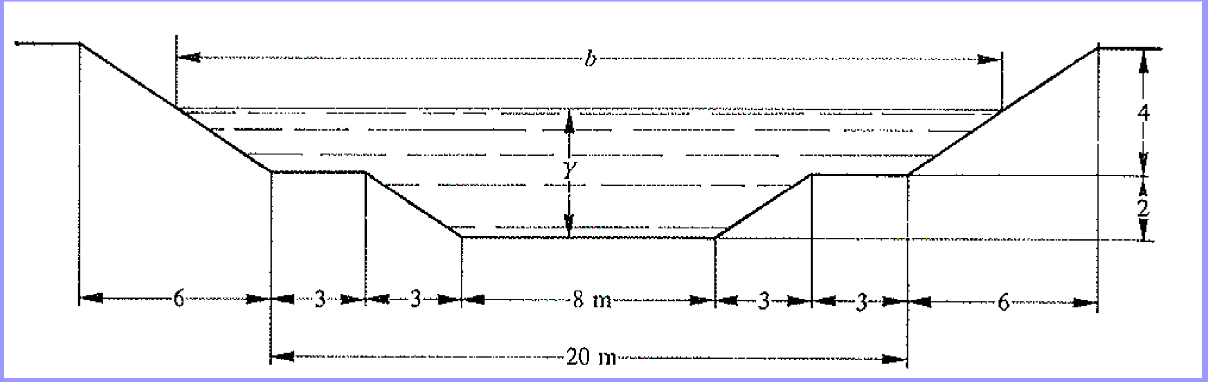
$$Q = k_s R(h)^{1/6} A(h) \sqrt{R(h) S_b}$$

3

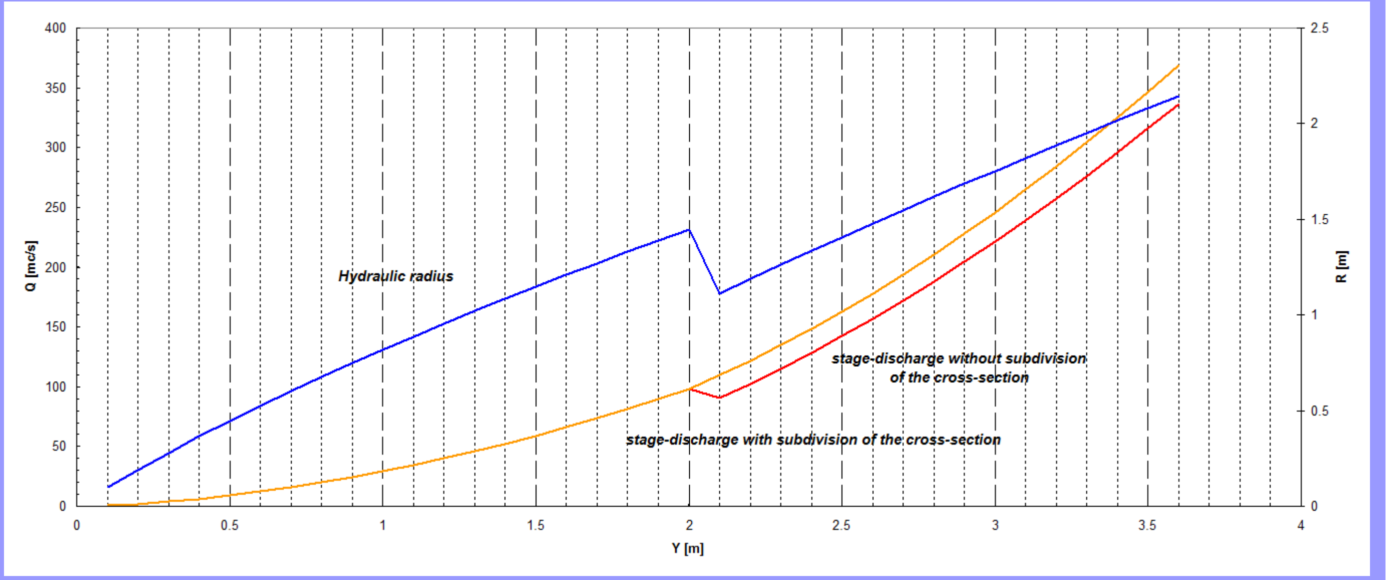


OPEN CHANNEL FLOW: uniform motion

Often natural or man-made cross section are composite, i.e., composed of different subsections, maybe with different roughness and local slope, due to different lengths of the thalweg. The lower subsection (alveo di magra) conveys water during drought or low flows. The overflow sections (alvei di piena e golenale) are activated during floods



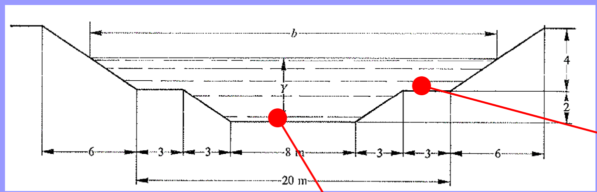
Without a proper decomposition the 1D assumption is violated and the hydraulic radius shows sudden reductions that have unrealistic effects on the other hydraulic quantities



HOMEWORK:
Give a careful look to the spreadsheet Compound_Cross_Section.xls

OPEN CHANNEL FLOW: uniform motion

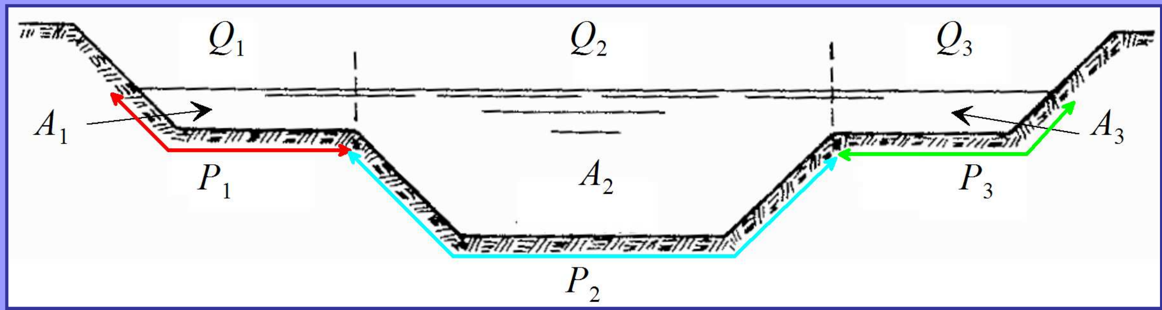
“Zona golenale” of the Po river at Isola Pescaroli (floodplain)



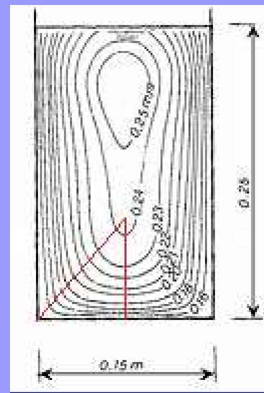
“Alveo di magra” (main bed or channel)

OPEN CHANNEL FLOW: uniform motion in channels of compound section

The cross section of a channel may be composed of several subsections, e.g. a main channel and two side channels (flood plains). In this case the Chézy equation has to be applied to each subsection *i* to compute the corresponding discharge Q_i . The total discharge is obtained as $Q=\Sigma Q_i$



For the evaluation of the wetted perimeter P_i of each subsection <u>only the solid boundaries</u> are considered. This criterion would require to subdivide the section along the lines orthogonal to the isotachs; actually, along these lines no internal shear stress takes place; however, vertical lines are generally used. The hydraulic radius R_i of each subsection is calculated as $R_i=A_i/P_i$.

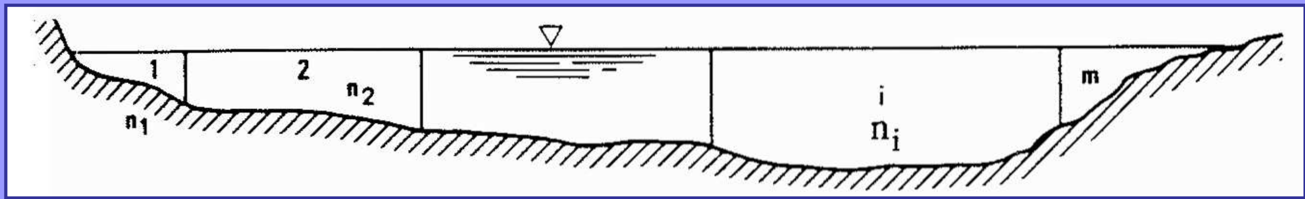


$$Q = \sum_i Q_i = \sum_i \frac{1}{n_i} R_i^{2/3} A_i S_b^{1/2} = K S_b^{1/2}, \quad K = \sum_i \frac{1}{n_i} R_i^{2/3} A_i$$

conveyance

OPEN CHANNEL FLOW: equivalent roughness

In case of ***compact sections*** in which the roughness may be different from part to part of the perimeter the discharge can be computed without actually subdividing the section. To this purpose, an equivalent roughness coefficient can be introduced dividing the water area into *N* parts of which the wetted perimeter P_i (<u>calculated taking into account only the solid boundaries</u>) and roughness coefficients n_i are known.



Assuming the same mean velocity for each partial area, in uniform flow (according to Horton and Einstein)

$$V = \frac{1}{\bar{n}} R^{2/3} S_b^{\,1/2} \quad \Rightarrow \quad \frac{V^{3/2}}{S_b^{\,3/4}} = \frac{1}{\bar{n}^{3/2}} \frac{A}{P} = \frac{1}{n_i^{\,3/2}} \frac{A_i}{P_i} = \frac{V_i^{\,3/2}}{S_b^{\,3/4}} \quad \Rightarrow \quad \frac{1}{\bar{n}^{3/2}} \frac{A}{P} P_i n_i^{\,3/2} = A_i$$

from which

$$\frac{1}{\bar{n}^{3/2}} \frac{A}{P} \sum_i P_i n_i^{\,3/2} = A \quad \Rightarrow \quad \boxed{\bar{n} = \left(\frac{\sum_i P_i n_i^{\,3/2}}{P} \right)^{2/3} \qquad \left(P = \sum P_i\right)}$$

$$k_s = \frac{\sum \left(P_i k_{si} R_i^{5/3}\right)}{P R^{5/3}}$$

In a similar way, Pavloskii and Einstein, considering the tractive force along the boundary as the sum of the single contributions

$$k_s = \frac{P^{0.5}}{\left(\sum \frac{P_i}{k_{si}^2} \right)^{0.5}}$$

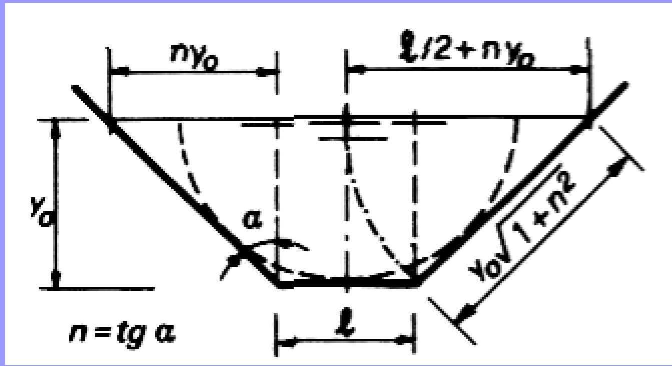
And Lotter, regarding the overall discharge as the sum of the single contributions

HOMEWORK:
Compare these relationship by using a spreadsheet

Section of Maximum discharge

$$Q = k_s \frac{A(h)^{5/3}}{P(h)^{2/3}}\sqrt{S_b} = C\frac{A(h)^{5/3}}{P(h)^{2/3}}$$

Let us suppose that Q, ks and S_b are kept constant. The channel will be less expensive if A is the smallest possible. This condition is equivalently sattisfied if, for A given, P is minimum.



Let us consider a trapezoidal cross-section, that is a function of l, y_o and n. If A is kept constant, these three variables are constrained, because

$$A(h) = (l+nh)h; \quad l = \frac{A(h)}{h} - nh$$

The perimeter is given by $P(h) = l + 2h\sqrt{1+n^2} = \frac{A}{h} - nh + 2h\sqrt{1+n^2}$

And, depending on the quantities that can be varied in our problem, can be differentiated either with rispect to n or to h. If one differentiate with respect to h

$$dP = -\frac{A}{h^2} - n + 2\sqrt{1+n^2} = 0; \quad 2\sqrt{1+n^2} = n + \frac{A}{h^2}$$

So that the perimeter is

$$P(h) = l + hn + \frac{A}{h} = 2\frac{A}{h} \qquad \boxed{R(h) = \frac{A}{P} = \frac{h}{2}}$$

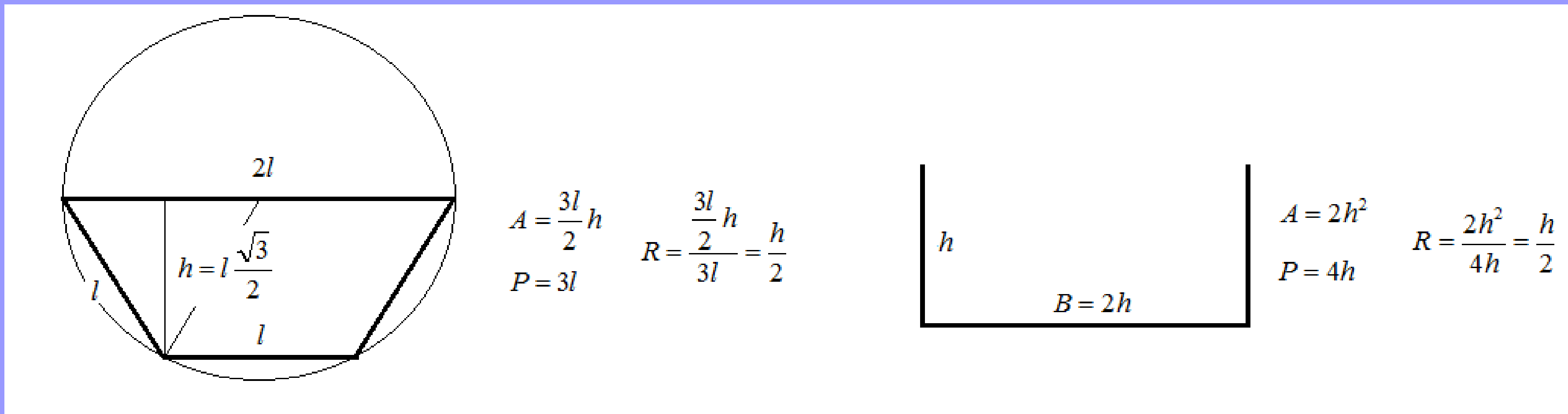
Which is the condition to be sattisfied in order to minimize the area. This condition is true indipendently from n. If n =0 we have an optimal rectangular cross section for B=2h

OPEN CHANNEL FLOW: uniform motion

$$dP = -h + \frac{2hn}{\sqrt{1+n^2}} = 0; \qquad \sqrt{1+n^2} = 2n$$

Always with A constant, If one can minimize also with respect to n one obtains this condition that corresponds to n=tg30°

These two conditions provide an additional constraint to identify the optimal cross section
Accordingly, if everything can be chosen, an half exagonal cross section seems to be the most reasonable.
On the other hand, if the choice is constrained to a rectangular cross-section, B=2h provides the best choice.



Whether this choice is practicable or not depends on other constraints, such as, for instance, the type of lining used to cover the channel surface, the actual availability of space around the channel or the maximum allowable velocity.

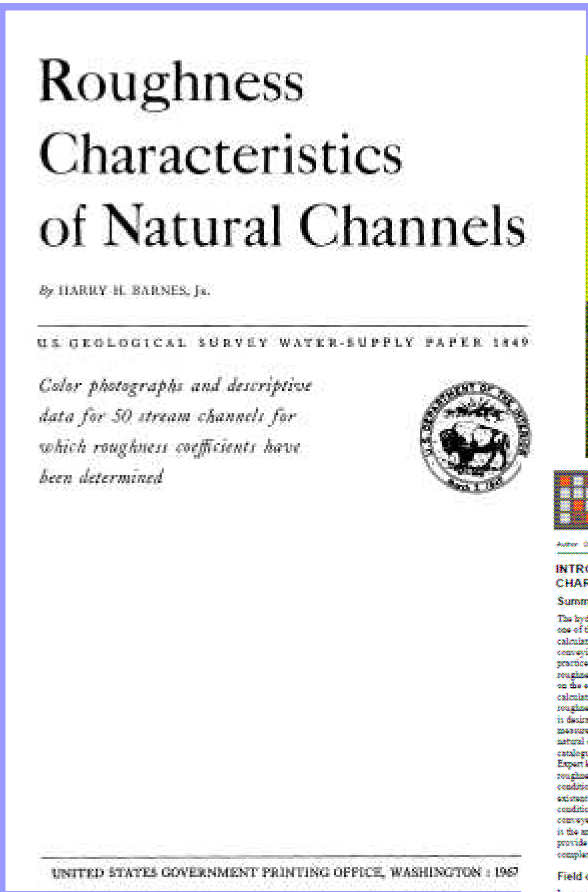
OPEN CHANNEL FLOW: uniform motion and selection of roughness coefficient

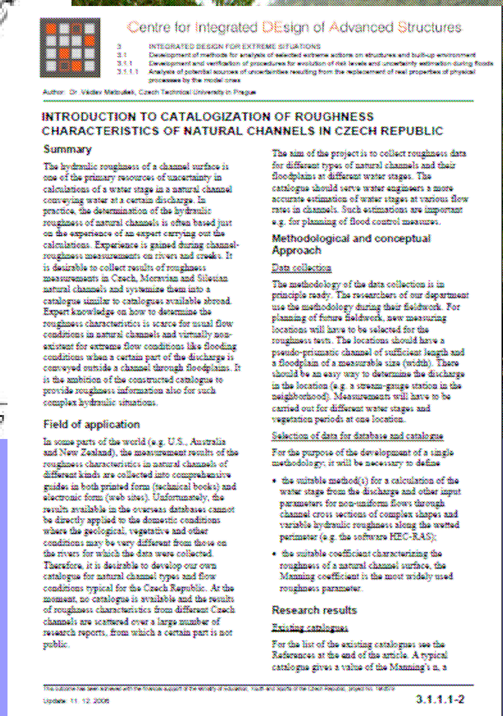
Tables of n values for channels of various kinds can be found in the literature (e.g. Chow, 1964)

Type of channel and description	Minimum	Normal	Maximum
D. Natural Streams			
D-1. Minor streams (top width at flood stage <100 ft)			
a. Streams on plain			
1. Clean, straight, full stage, no rifts or deep pools	0.025	**0.030**	0.033
2. Same as above, but more stones and weeds	0.030	0.035	0.040
3. Clean, winding, some pools and shoals	0.033	0.040	0.045
4. Same as above, but some weeds and stones	0.035	0.045	0.050
5. Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
6. Same as 4, but more stones	0.045	0.050	0.060
7. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
8. Very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150

Type of channel and description	Minimum	Normal	Maximum
b. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
1. Bottom: gravels, cobbles, and few boulders	0.030	0.040	0.050
2. Bottom: cobbles with large boulders	0.040	0.050	0.070
D-2. Flood plains			
a. Pasture, no brush			
1. Short grass	0.025	0.030	0.035
2. High grass	0.030	0.035	0.050
b. Cultivated areas			
1. No crop	0.020	0.030	0.040
2. Mature row crops	0.025	0.035	0.045
3. Mature field crops	0.030	0.040	0.050
c. Brush			
1. Scattered brush, heavy weeds	0.035	0.050	0.070
2. Light brush and trees, in winter	0.035	0.050	0.060
3. Light brush and trees, in summer	0.040	0.060	0.080
4. Medium to dense brush, in winter	0.045	0.070	0.110
5. Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
1. Dense willows, summer, straight	0.110	0.150	0.200
2. Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
3. Same as above, but with heavy growth of sprouts	0.050	0.060	0.080
4. Heavy stand of timber, a few down trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
5. Same as above, but with flood stage reaching branches	0.100	0.120	0.160
D-3. Major streams (top width at flood stage >100 ft). The *n* value is less than that for minor streams of similar description, because banks offer less effective resistance.			
a. Regular section with no boulders or brush	0.025	.....	0.060
b. Irregular and rough section	0.035	.....	0.100

OPEN CHANNEL FLOW: uniform motion and selection of roughness coefficient





Centre for Integrated DEsign of Advanced Structures

3 INTEGRATED DESIGN FOR EXTREME SITUATIONS
3.1 Development of methods for analysis of selected extreme actions on structures and built-up environment
3.1.1 Development and verification of procedures for evolution of risk levels and uncertainty estimation during floods
3.1.1.1 Analysis of potential sources of uncertainties resulting from the replacement of real properties of physical processes by the model ones

Author: Dr. Václav Matoušek, Czech Technical University in Prague

INTRODUCTION TO CATALOGIZATION OF ROUGHNESS CHARACTERISTICS OF NATURAL CHANNELS IN CZECH REPUBLIC

Summary

The hydraulic roughness of a channel surface is one of the primary resources of uncertainty in calculations of a water stage in a natural channel conveying water at a certain discharge. In practice, the determination of the hydraulic roughness of natural channels is often based just on the experience of an expert carrying out the calculations. Experience is gained during channel-roughness measurements on rivers and creeks. It is desirable to collect results of roughness measurements in Czech, Moravian and Silesian natural channels and systemize them into a catalogue similar to catalogues available abroad. Expert knowledge on how to determine the roughness characteristics is scarce for usual flow conditions in natural channels and virtually non-existent for extreme flow conditions like flooding conditions when a certain part of the discharge is conveyed outside a channel through floodplains. It is the ambition of the constructed catalogue to provide roughness information also for such complex hydraulic situations.

Field of application

In some parts of the world (e.g. U.S., Australia and New Zealand), the measurement results of the roughness characteristics in natural channels of different kinds are collected into comprehensive guides in both printed form (technical books) and electronic form (web sites). Unfortunately, the results available in the overseas databases cannot be directly applied to the domestic conditions where the geological, vegetative and other conditions may be very different from those on the rivers for which the data were collected. Therefore, it is desirable to develop our own catalogue for natural channel types and flow conditions typical for the Czech Republic. At the moment, no catalogue is available and the results of roughness characteristics from different Czech channels are scattered over a large number of research reports, from which a certain part is not public.

The aim of the project is to collect roughness data for different types of natural channels and their floodplains at different water stages. The catalogue should serve water engineers a more accurate estimation of water stages at various flow rates in channels. Such estimations are important e.g. for planning of flood control measures.

Methodological and conceptual Approach

Data collection

The methodology of the data collection is in principle ready. The researchers of our department use the methodology during their fieldwork. For planning of future fieldwork, new measuring locations will have to be selected for the roughness tests. The locations should have a pseudo-prismatic channel of sufficient length and a floodplain of a measurable size (width). There should be an easy way to determine the discharge in the location (e.g. a stream-gauge station in the neighborhood). Measurements will have to be carried out for different water stages and vegetation periods at one location.

Selection of data for database and catalogue

For the purpose of the development of a single methodology, it will be necessary to define

- the suitable method(s) for a calculation of the water stage from the discharge and other input parameters for non-uniform flows through channel cross sections of complex shapes and variable hydraulic roughness along the wetted perimeter (e.g. the software HEC-RAS);
- the suitable coefficient characterizing the roughness of a natural channel surface, the Manning coefficient is the most widely used roughness parameter.

Research results

Existing catalogues

For the list of the existing catalogues see the References at the end of the article. A typical catalogue gives a value of the Manning's n, a

Update: 11. 12. 2006

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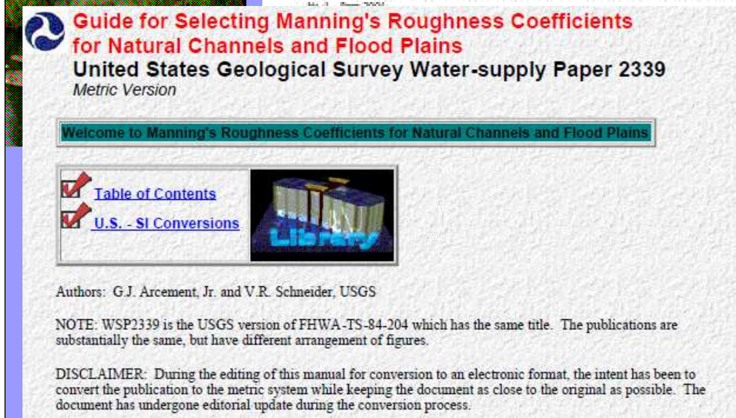


Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains
United States Geological Survey Water-supply Paper 2339
Metric Version

Welcome to Manning's Roughness Coefficients for Natural Channels and Flood Plains

Table of Contents
U.S. - SI Conversions

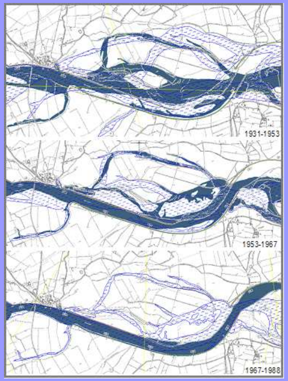
Authors: G.J. Arcement, Jr. and V.R. Schneider, USGS

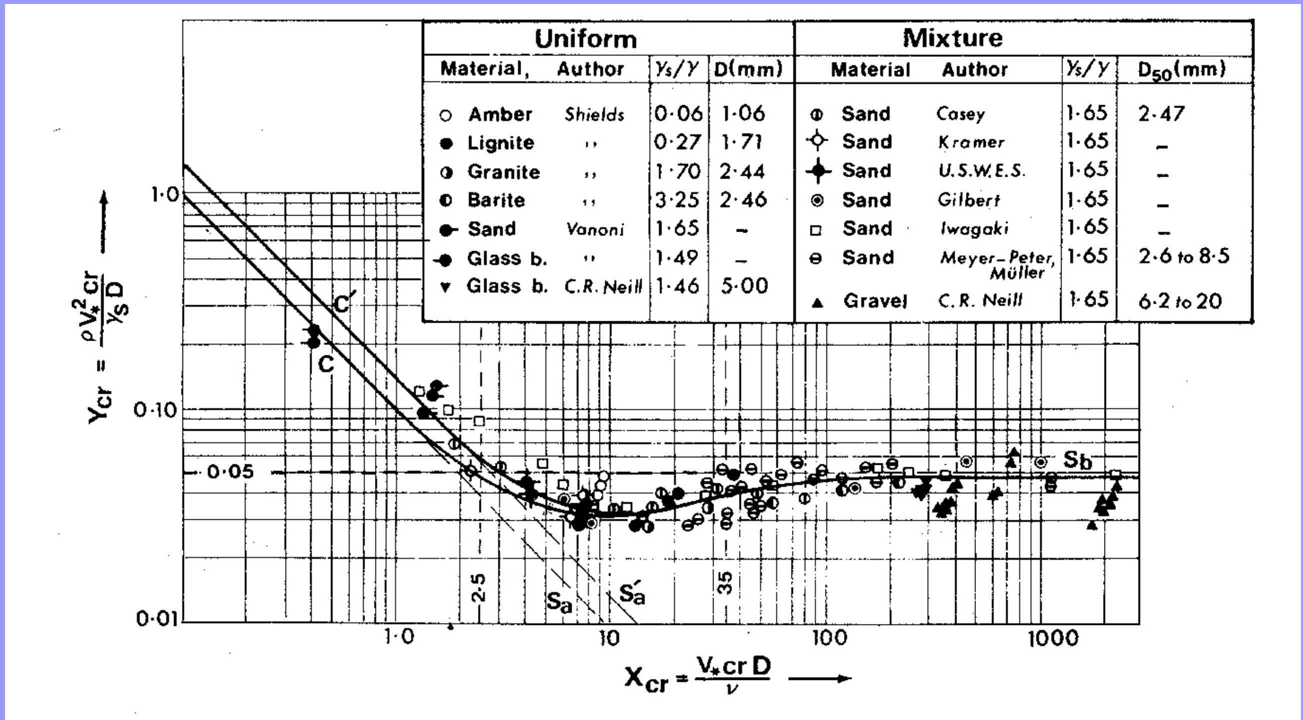
NOTE: WSP2339 is the USGS version of FHWA-TS-84-204 which has the same title. The publications are substantially the same, but have different arrangement of figures.

DISCLAIMER: During the editing of this manual for conversion to an electronic format, the intent has been to convert the publication to the metric system while keeping the document as close to the original as possible. The document has undergone editorial update during the conversion process.

OPEN CHANNEL FLOW: a remark on the applicability of the fixed bed hypothesis

The hypothesis of unerodible and fixed bed is true whenever the flow lies below the Shields diagram : $Y < Y_c$





Where $Y = \dfrac{\rho v_*^2}{\gamma_s D} = \dfrac{\tau_0}{\gamma_s D}$ *mobility number [-], with γ_s sediment submerged specific weight [N/m^3]*

$X = \dfrac{\rho v_* D}{\mu}$ *grain Reynolds number [-]*

For a better definition of the left side of the Shields diagram see
Pilotti M., Menduni G., Beginning of sediment transport of incoherent grains in shallow shear flows, Journal of Hydraulic Research, IAHR, 39, 115-124, 2001.

OPEN CHANNEL FLOW: Specific Energy

$$\boxed{E = h + \frac{\alpha U^2}{2g} = h + \frac{\alpha Q^2}{2gA(h)^2}}$$

(G.2b) Specific Energy with respect to the thalweg, with Q constant

$$\frac{dE}{dh} = 1 - \frac{\alpha Q^2}{gA(h)^3}\frac{dA}{dh} = 1 - \frac{\alpha U^2}{g\bar{h}} = 1 - \alpha Fr^2 = 0$$

(E.1) Minimum of E(h)

$$\bar{h} = \frac{A(h)}{B(h)}$$

(E.2) Equivalent (average) Hydraulic Depth

$$\boxed{1 = \frac{\alpha Q^2}{gA(k)^3} B(k)}$$

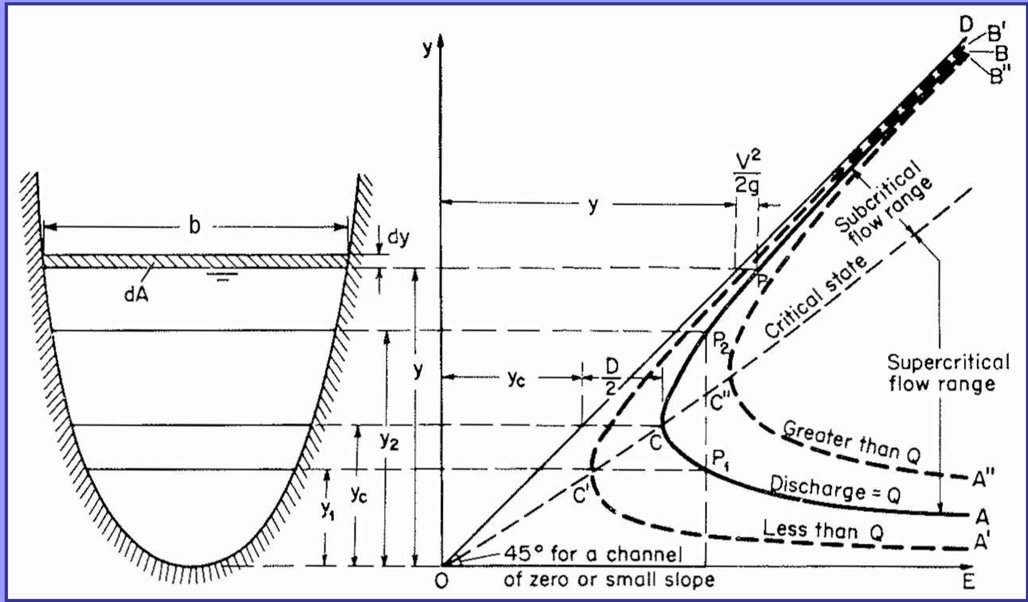
(E.3) General expression for critical depth

$$k = \sqrt[3]{\frac{\alpha Q^2}{gB^2}}$$

(E.4) Critical depth in rectangular channel

$$E_k = k + \frac{\bar{k}}{2}$$

General espression for Specific Energy in critical Condition

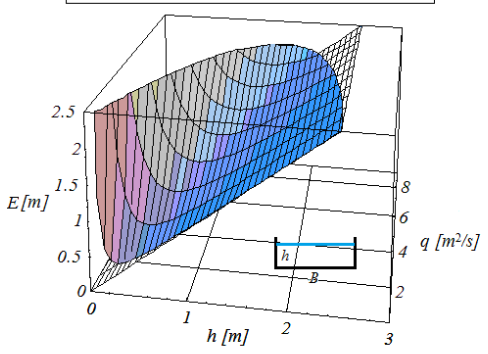


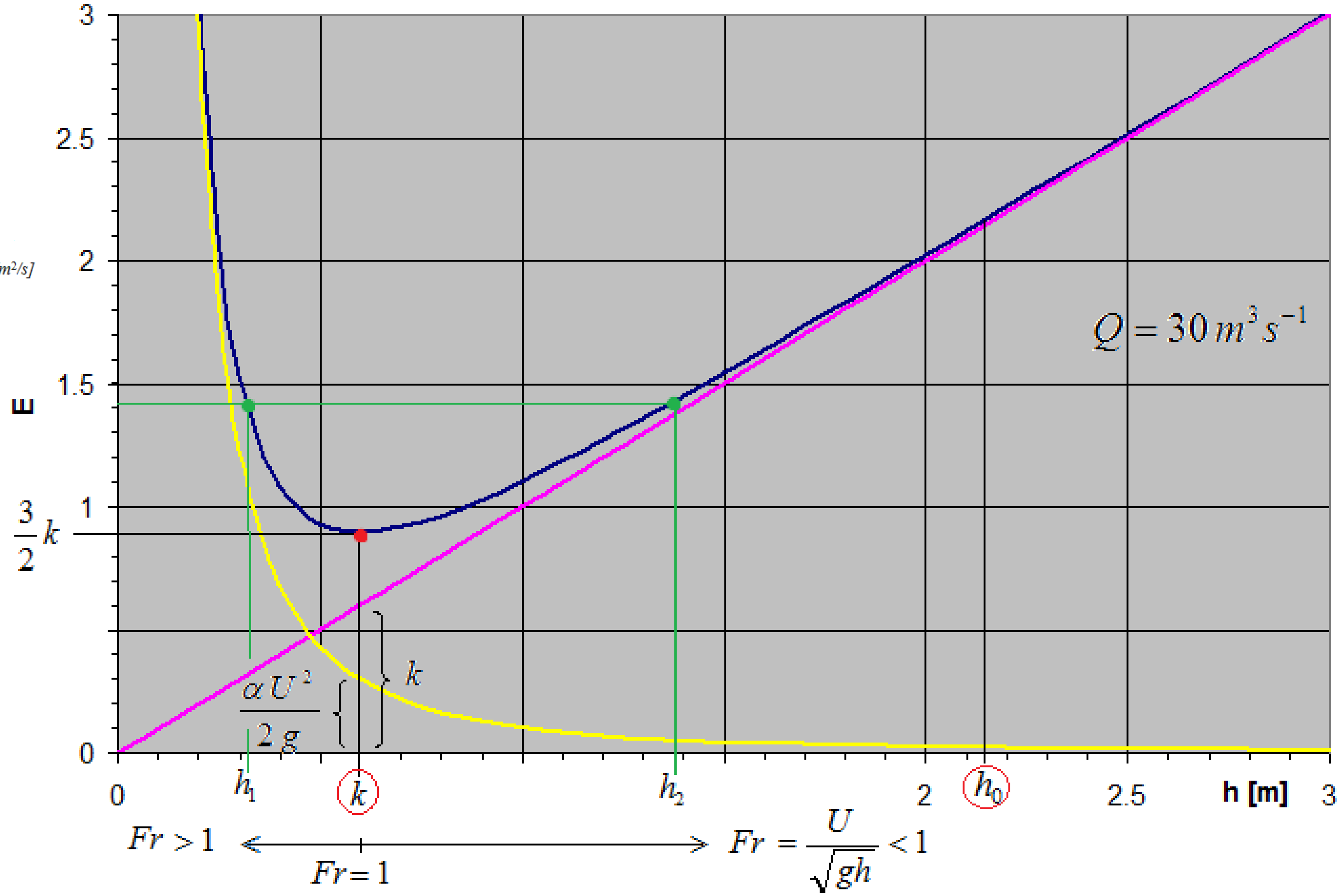
$$k = \frac{2}{3}E_k\,; \quad k = \frac{4}{5}E_k\,; \quad k = \frac{3}{5}E_k$$

E(k) in Rectangular, Triangular and Parabolic cross-section;

OPEN CHANNEL FLOW: Specific Energy

$$E = h + \alpha\frac{U^2}{2g} = h + \alpha\frac{Q^2}{2gB^2h^2} = h + \alpha\frac{q^2}{2gh^2}$$





For a given channel section and a given discharge the critical depth yc depends only on the geometry of the section, while the normal depth $h = h_0$ depends on the slope of the channel and the roughness coefficient.

Depending on the relative position between h_0 and k, the bottom slope is defined as

Mild slope: $h_0 > k$ (see figure above); Steep slope: $h_0 < k$; Critical slope: $h_0 = k$

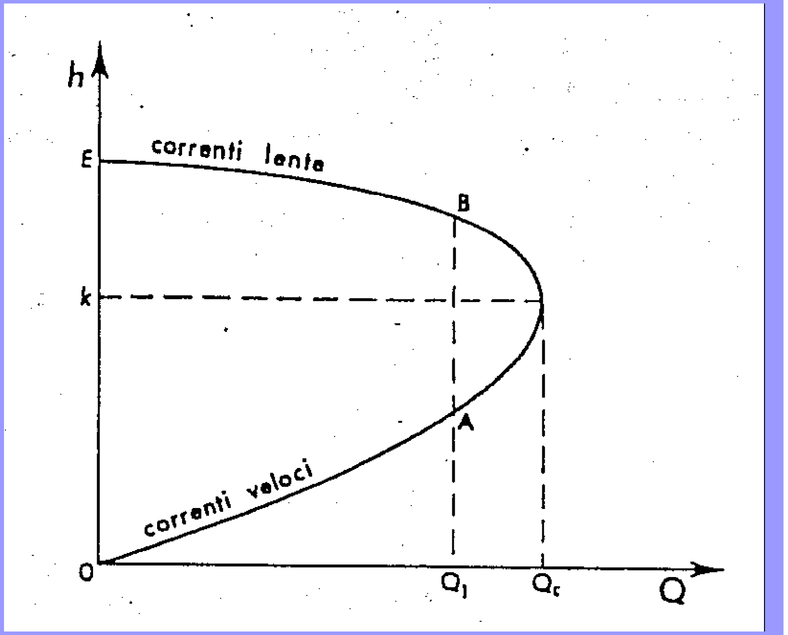
$$Q = A(h)\sqrt{\frac{2g}{\alpha}(E-h)}$$

Specific discharge for E constant

$$\frac{dQ}{dh} = 0 \quad \rightarrow \quad E = h + \frac{\overline{h}}{2} \quad \rightarrow h \equiv k$$

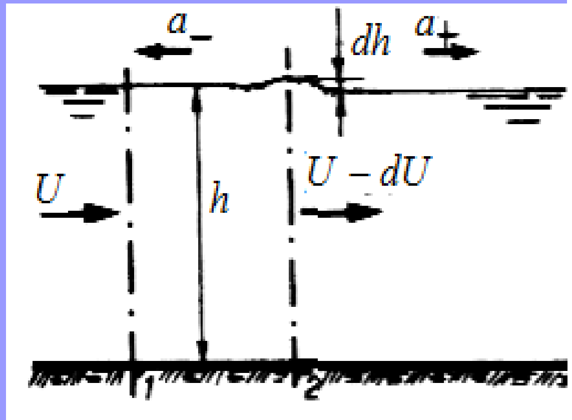
$$\frac{Q}{B} = q = h\sqrt{\frac{2g}{\alpha}(E-h)}$$

In a rectangular cross section



OPEN CHANNEL FLOW: Froude number

Let us underline the meaning of the Froude number.
Let us consider an infinitely wide channel where water flows in uniform motion with depth h and velocity U.
If perturbation affects the whole water column (tsunami like), we have a wave of positive height dh that may travel upstream and downstream with absolute celerity ±a. Due to its passage U is modified, as U-dU.
Given that the motion is an unsteady one, it is convenient to study the process as seen from astride the wave. This is a inertial frame of reference so that both energy and mass balance can be written in terms of relative velocity.
We can write



$$\vec{v}_a = \vec{v}_r + \vec{v}_t$$

$$U = U_r + a; \qquad U_r = U - a$$

$$\frac{dE}{ds} = 0 \qquad h + \frac{(U-a)^2}{2g} = (h + dh) + \frac{(U - dU - a)^2}{2g} \qquad dh = dU\frac{(U-a)}{g}$$

energy balance

$$\frac{dQ}{ds} = 0 \qquad (U-a)h = (U - dU - a)(h + dh) \qquad dU = dh\frac{(U-a)}{h}$$

mass balance

$$a = U \mp \sqrt{gh} = U \mp c = \sqrt{gh}\,(Fr \mp 1)$$

Let us observe our final result

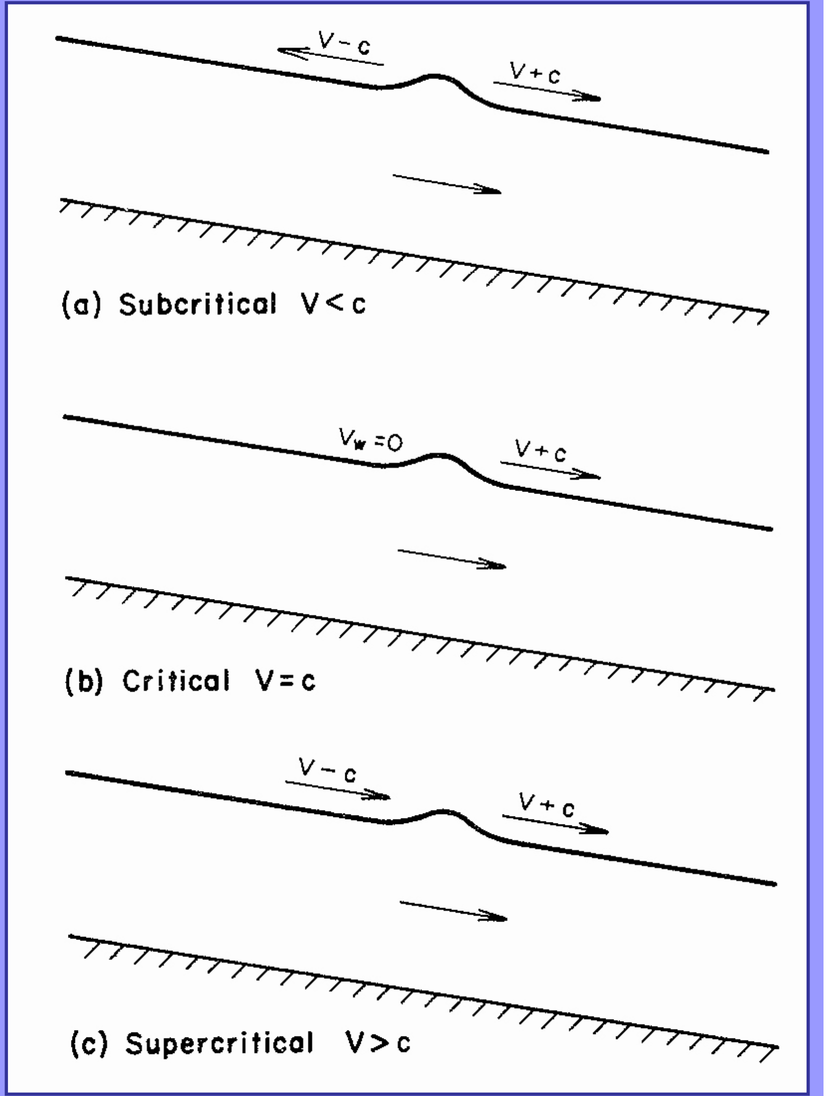
$$a = U \mp \sqrt{gh} = U \mp c = \sqrt{gh}\,(Fr \mp 1)$$

Where c is the wave relative celerity according to Lagrange (1788).

If we consider the case when a is positive, there are two values of a >0 only if Fr >1; otherwise only one is positive. If we consider the case when a is negative, there is only one possible value of a <0 when Fr <1;

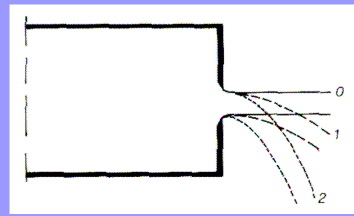
Accordingly: if Fr < 1 then there is a positive value of a and a negative one and every perturbation can move both upstream and downstream.
if Fr > 1 both values of a are positive, so that the wave cannot propagate upstream.
if Fr = 1, c=U (see previous slides) and a = 0

Note that c is generally different from U. The infinitely small wave propagates with a celerity that is different from the average mass velocity, U.



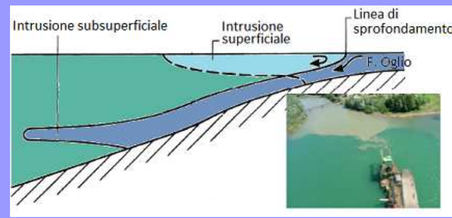
$$\frac{\rho v \nabla v V}{\rho g V} \propto \frac{\frac{\rho U^2}{L}}{\rho g} = \frac{U^2}{gL} = Fr^2$$

Froude number can be introduced when studying jets, as the ratio between inertial and gravitational Forces



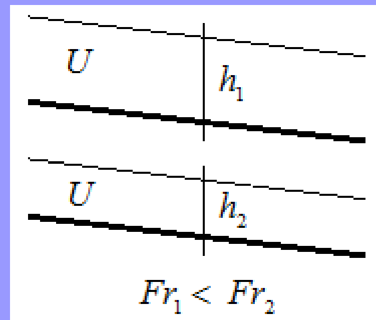
$$\frac{\rho v \nabla v V}{(\rho - \rho_1) g V} \propto \frac{\frac{\rho U^2}{L}}{\frac{\Delta \rho}{\rho} \rho g} = \frac{U^2}{\frac{\Delta \rho}{\rho} gL} = Fr_d^{\,2}$$

In general terms it should take into account the density of the fluid where the jet is taking place (densimetric Froude number)



$$\frac{\frac{U^2}{2g}}{\frac{p}{\gamma}} = \frac{\frac{U^2}{2g}}{\frac{\gamma h}{\gamma}} = \frac{1}{2}\frac{U^2}{gh} = \frac{1}{2} Fr^2$$

But also in open channel flow as the semi-ratio between the kinetic energy per unit weight over the energy related to pressure (after Bakhmeteff, 1912).



$$\frac{D\vec{v}^*}{D\tau} = \frac{1}{Fr^2}\nabla z^* - \nabla p^* + \frac{1}{\mathrm{Re}}\Delta \vec{v}^*$$

$$\vec{v}^* = f(x^*, \tau, Fr, \mathrm{Re});$$

$$p^* = f(x^*, \tau, Fr, \mathrm{Re})$$

Finally it arises when Navier Stokes equations are made dimensionless. This result is particularly important because it dictates the Froude similarity criterion. Accordingly, if Froude similarity is imposed and one defines $\lambda_l = l/L$, $\lambda_t = t/T$ and $\lambda_v = v/V$, then the constraints hold

$$\lambda_t = \sqrt{\lambda_l}; \quad \lambda_v = \sqrt{\lambda_l}$$

OPEN CHANNEL FLOW: steady flow profiles in prismatic channels

Let us consider a gradually varied flow, i.e. one in which vertical acceleration on the cross section are negligeable, and, accordingly, an hydrostatic pressure distribution is present. This happens if the slope of the channel is small and the geometry of the boundary is such that the streamlines are practically parallel. Under the above hypotheses, starting from energy equation of gradually varied flow.

$$\frac{dH}{dx} = -S_f$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(z + y + \frac{Q^2}{2gA^2}\right) = -S_0 + \frac{\partial E}{\partial y}\frac{dy}{dx} + \frac{\partial E}{\partial A}\frac{dA}{dx}$$

Let us now consider a prismatic channels, so that

$A=A(y(x))$ and S_b=constant

$$\frac{dy}{dx} = \frac{S_b - S_f}{dE/dy}, \quad \frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right) = 1 - \frac{Q^2 b}{gA^3} = 1 - Fr^2$$

$$\boxed{\frac{dy}{dx} = S_b \frac{1 - Q^2/(K^2 S_0)}{1 - Fr^2} = S_b \frac{1 - Q^2/Q_0^{\,2}}{1 - Fr^2}}$$

From the equation of gradually varied flow in a prismatic channel, the following general properties of the flow profile y(x) are easily obtained:

$$\boxed{\frac{dy}{dx}=\frac{N(y;Q)}{D(y;Q)},\quad N=S_b\left(1-\frac{Q^2}{Q_0^2}\right),\quad D=1-Fr^2}$$

$$y\to\infty \;\Rightarrow\; \begin{cases} Q_0\to\infty \\ Fr\to 0\end{cases} \;\Rightarrow\; \begin{cases} N\to 1 \\ D\to 1\end{cases} \;\Rightarrow\; \frac{dy}{dx}\to S_b$$

(asymptotic to a horizontal line)

$$y\to y_0 \;\Rightarrow\; Q_0\to Q \;\Rightarrow \begin{cases} N\to 0 \\ D\neq 0\end{cases} \;\Rightarrow\; \frac{dy}{dx}\to 0$$

(asymptotic to normal-depth line)

$$y\to y_c \;\Rightarrow\; Fr\to 1 \;\Rightarrow\; \begin{cases} N\neq 0 \\ D\to 0\end{cases} \;\Rightarrow\; \frac{dy}{dx}\to\infty$$

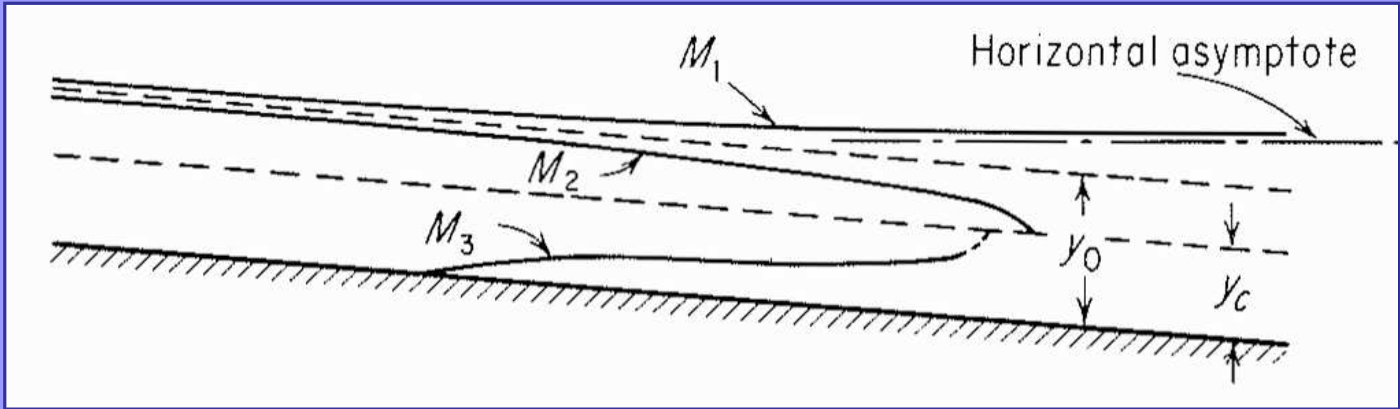
(asymptotic to a vertical line)

$$y\to 0 \;\Rightarrow\; \begin{cases} Q_0\to\infty \\ Fr\to\infty\end{cases} \;\Rightarrow\; \begin{cases} N\to -\infty \\ D\to -\infty\end{cases}$$

(dy/dx $\to\infty$ if Manning eq. Is used for Q_0)

Mild slope prismatic channel

$$dy/dx = N/D, \quad N = S_b\left(1 - Q^2/Q_0^2\right), \quad D = 1 - Fr^2$$

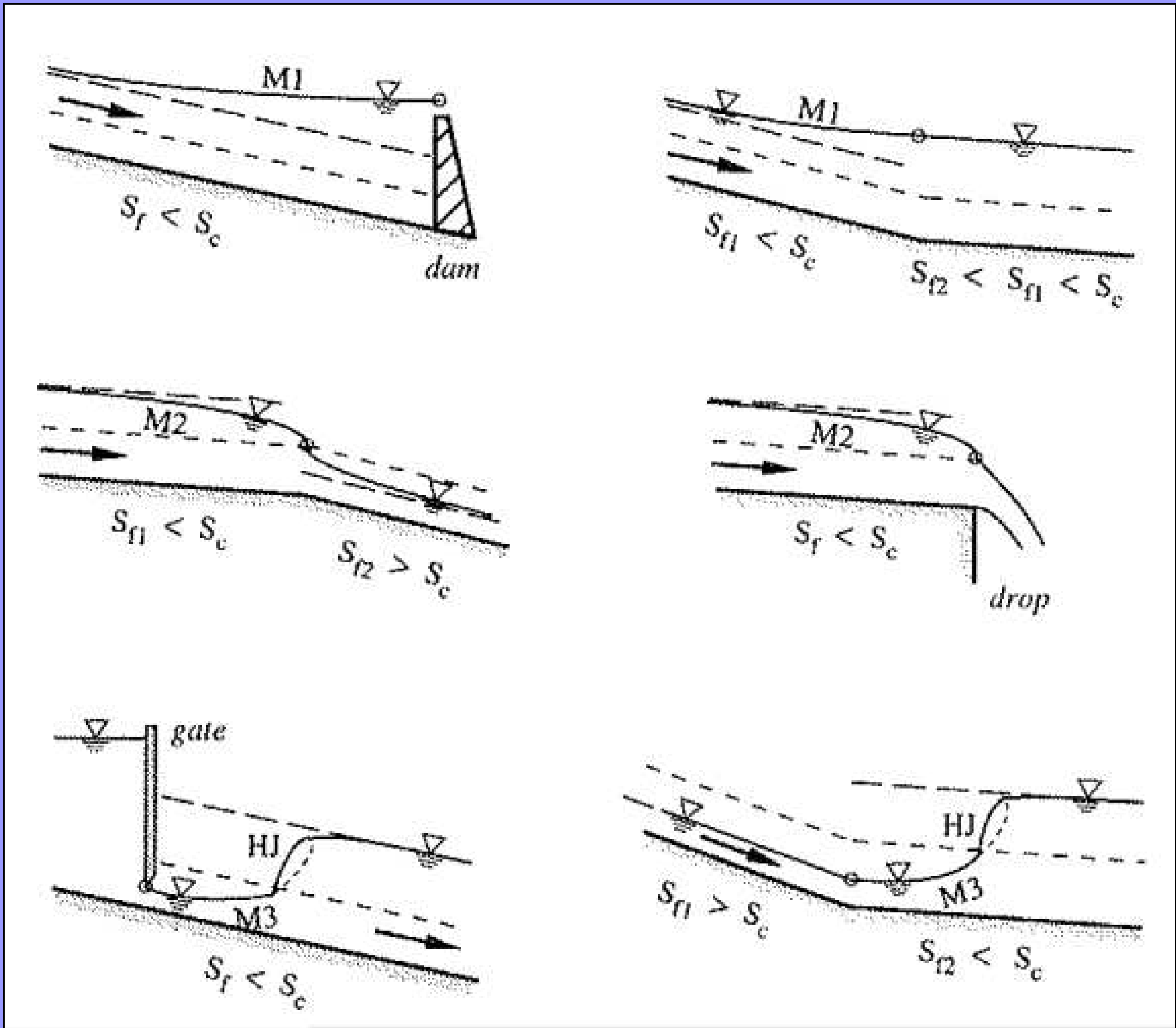


$$y > y_0 > y_c \quad \Rightarrow \quad \begin{cases} Q_0 > Q \\ Fr < 1 \end{cases} \Rightarrow \begin{cases} N > 0 \\ D > 0 \end{cases} \Rightarrow \frac{dy}{dx} > 0$$ *M1 profile*

$$y_c < y < y_0 \quad \Rightarrow \quad \begin{cases} Q_0 < Q \\ Fr < 1 \end{cases} \Rightarrow \begin{cases} N < 0 \\ D > 0 \end{cases} \Rightarrow \frac{dy}{dx} < 0$$ *M2 profile*

$$y < y_c < y_0 \Rightarrow \begin{cases} Q_0 < Q \\ Fr > 1 \end{cases} \Rightarrow \begin{cases} N < 0 \\ D < 0 \end{cases} \Rightarrow \frac{dy}{dx} > 0$$ *M3 profile*

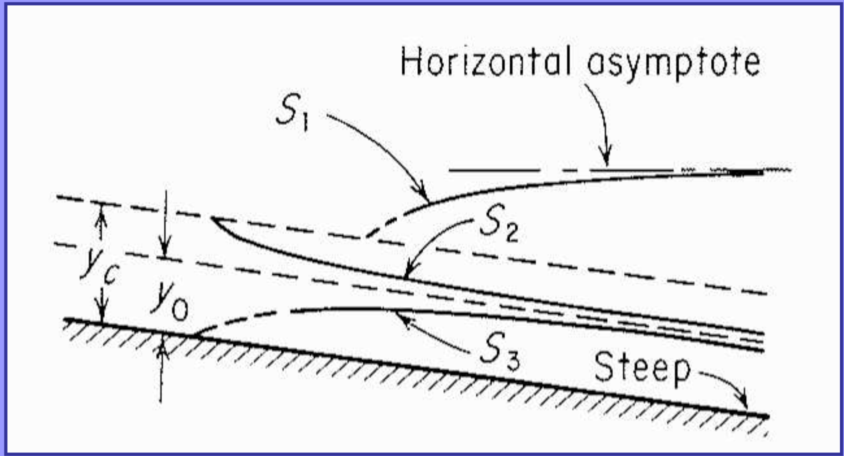
OPEN CHANNEL FLOW: steady flow profiles in mild slope prismatic channels



From W. H. Graf and M. S. Altinakar, 1998

Steep slope prismatic channel

$$dy/dx = N/D, \quad N = S_0\left(1 - Q^2/Q_0^2\right), \quad D = 1 - Fr^2$$



$$y > y_c > y_0 \quad \Rightarrow \quad \begin{cases} Q_0 > Q \\ Fr < 1 \end{cases} \quad \Rightarrow \quad \begin{cases} N > 0 \\ D > 0 \end{cases} \quad \Rightarrow \quad \frac{dy}{dx} > 0$$

S1 profile

$$y_0 < y < y_c \quad \Rightarrow \quad \begin{cases} Q_0 > Q \\ Fr > 1 \end{cases} \quad \Rightarrow \quad \begin{cases} N > 0 \\ D < 0 \end{cases} \quad \Rightarrow \quad \frac{dy}{dx} < 0$$

S2 profile

$$y < y_0 < y_c \quad \Rightarrow \quad \begin{cases} Q_0 < Q \\ Fr > 1 \end{cases} \quad \Rightarrow \quad \begin{cases} N < 0 \\ D < 0 \end{cases} \quad \Rightarrow \quad \frac{dy}{dx} > 0$$

S3 profile

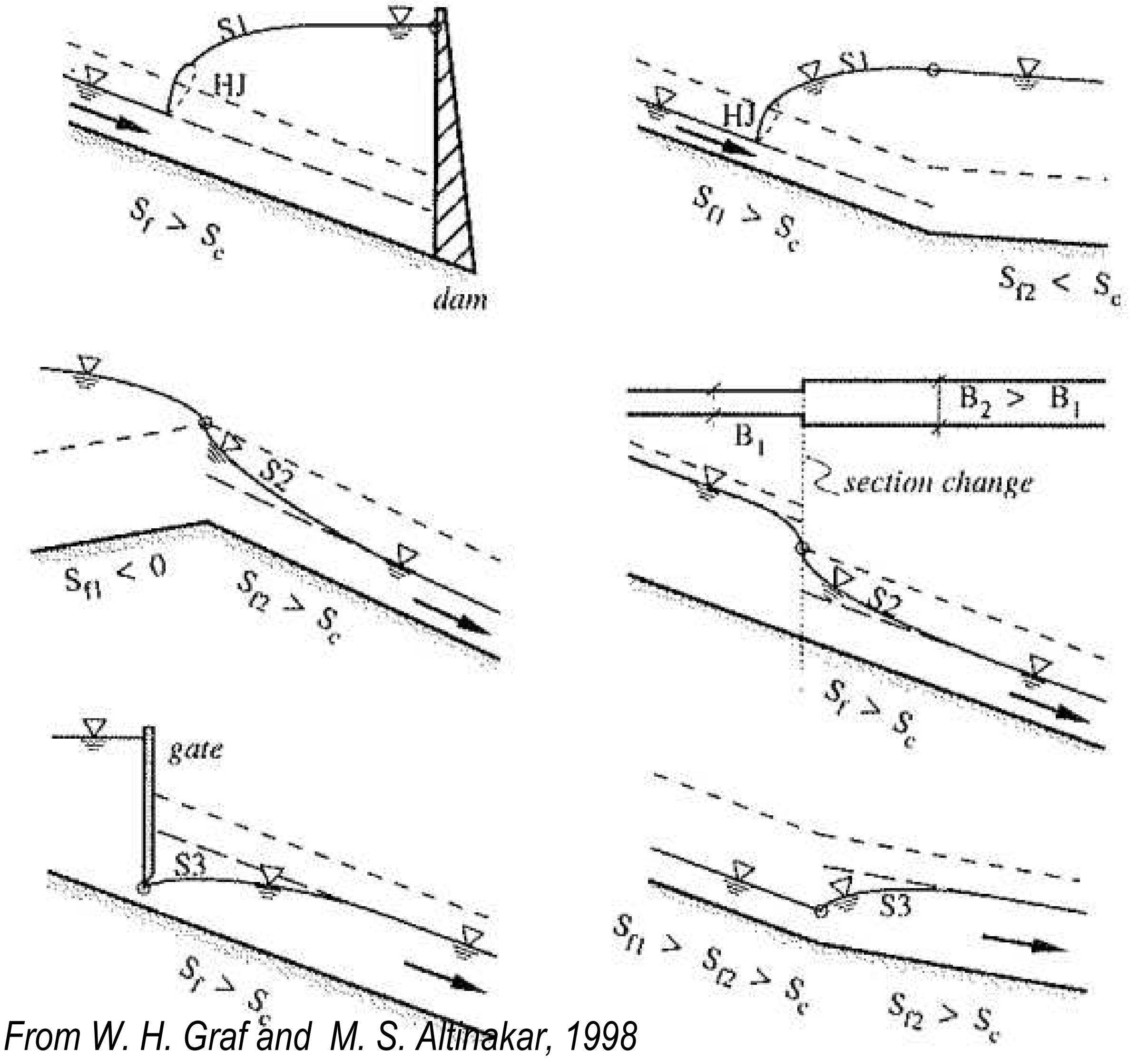
OPEN CHANNEL FLOW: steady flow profiles in steep slope prismatic channels

Boundary conditions:

Q known;

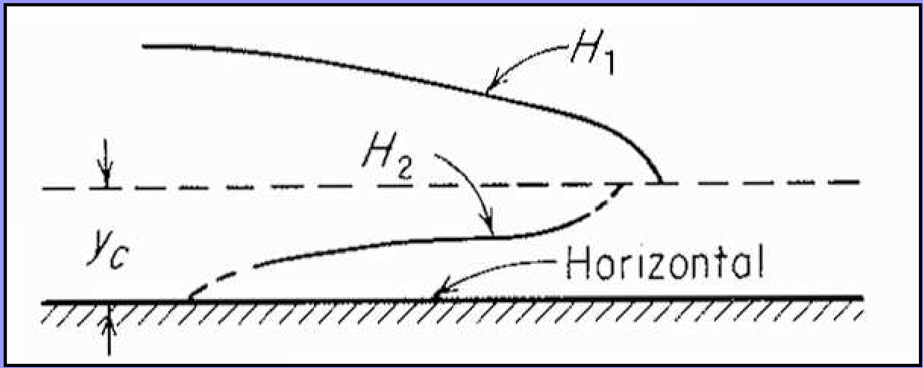
If Fr<1, Y downstream and the computation proceeds in the upstream direction along the channel.

If Fr>1, Y upstream and the computation proceeds in the downstream direction along the channel.

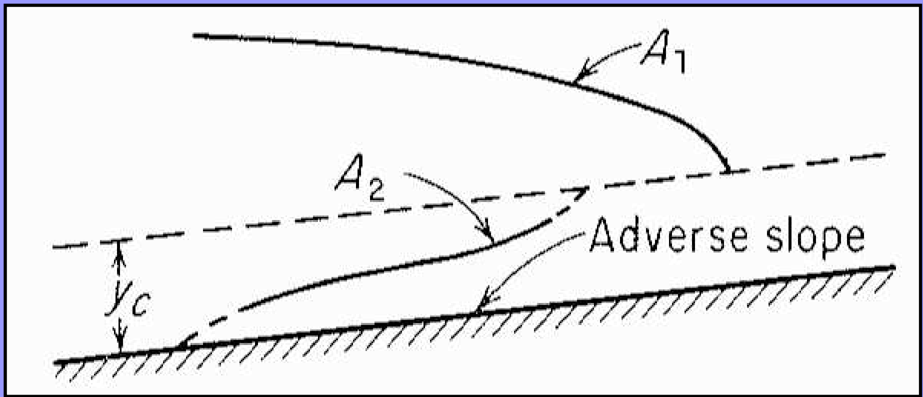


From W. H. Graf and M. S. Altinakar, 1998

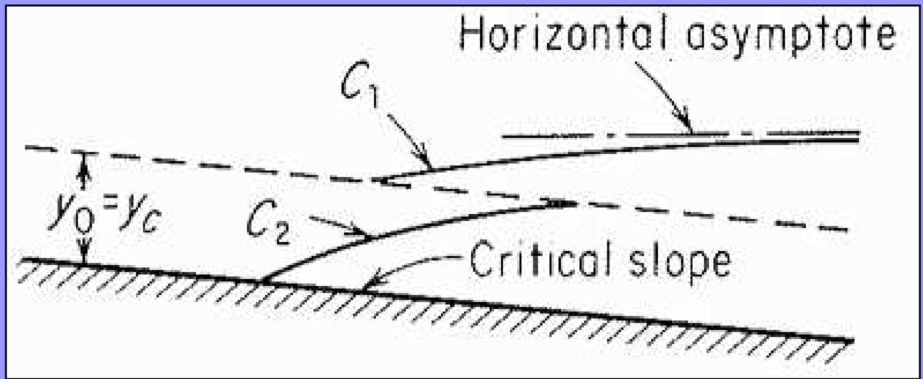
OPEN CHANNEL FLOW: steady flow profiles in prismatic channels



Horizontal slope prismatic channel

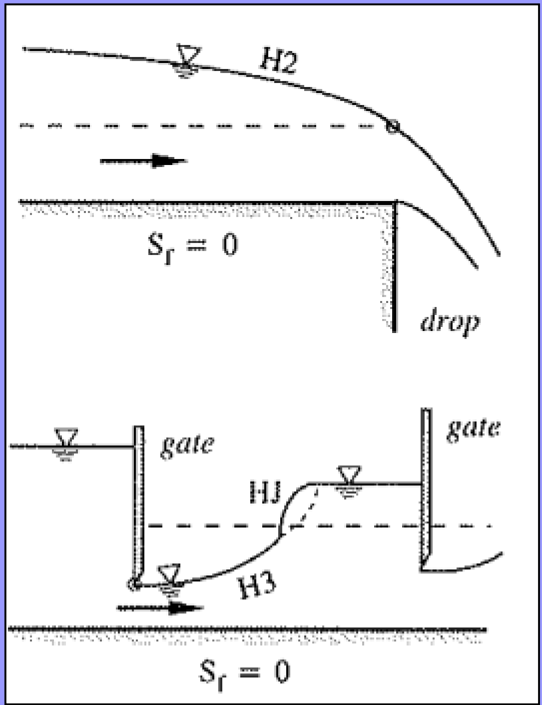


Adverse slope prismatic channel

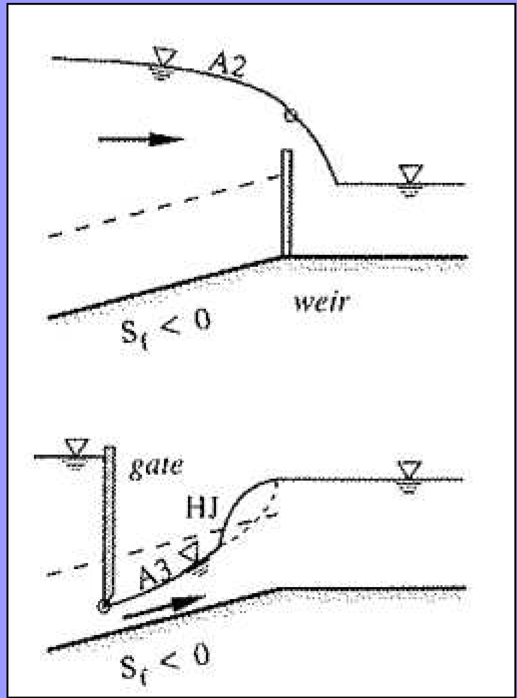


Critical slope prismatic channel

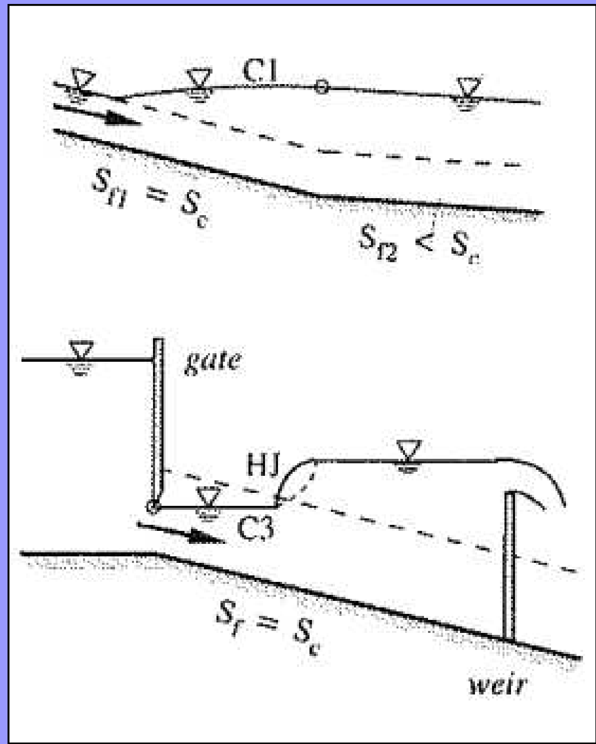
OPEN CHANNEL FLOW: steady flow profiles in various slope prismatic channels



Horizontal slope prismatic channel



Adverse slope prismatic channel



Critical slope prismatic channel

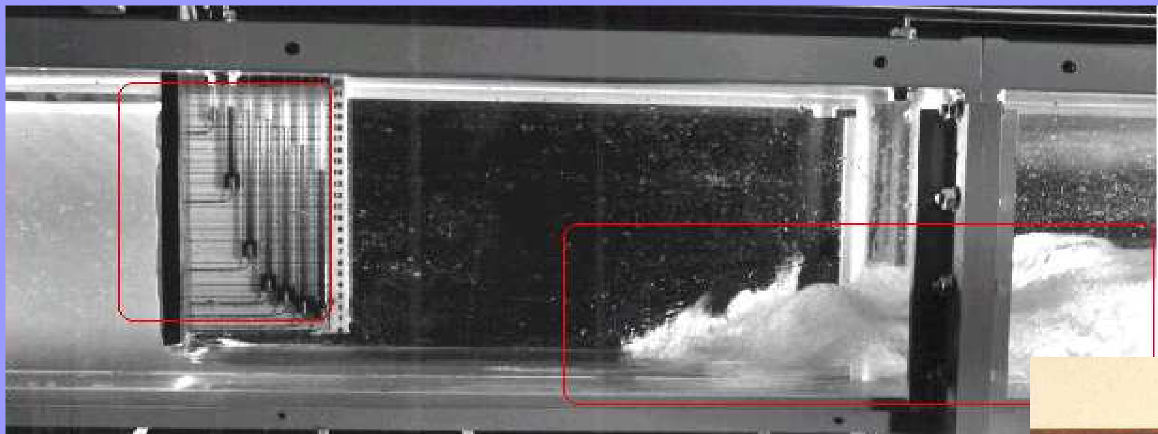
From W. H. Graf and M. S. Altinakar, 1998

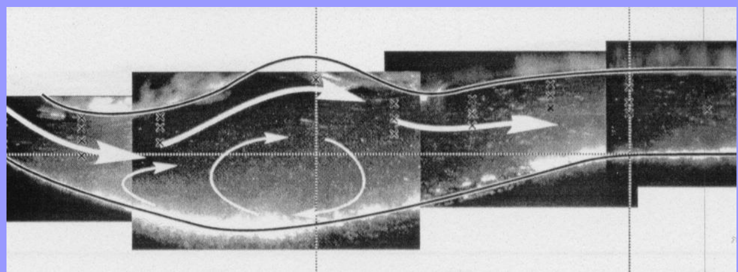
OPEN CHANNEL FLOW: hydraulic jump

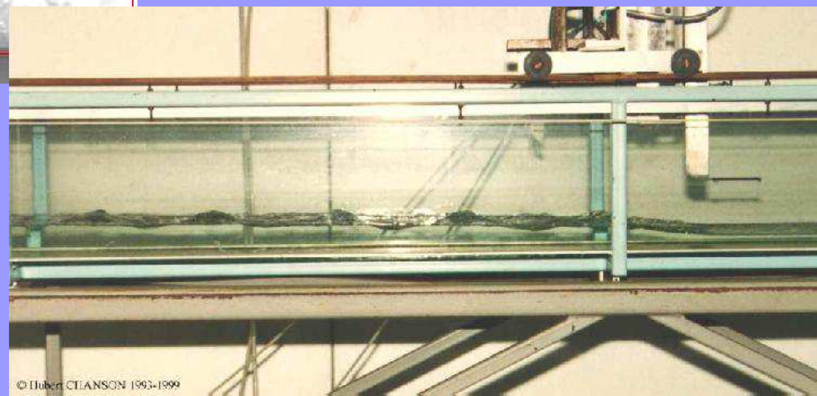
Supercritical flow in mild slope prismatic channels (M3 profile) and subcritical flow in steep slope prismatic channels (S1 profile) are limited downstream and upstream respectively at the critical depth. In these cases it may happen that supercritical flow has to be followed by subcritical flow to cover the whole channel length.

The change from supercritical to subcritical flow takes place abruptly through a vortex known as the hydraulic jump, characterized by considerable turbulence and energy loss.

The flow depths upstream and downsteam of the jump are called sequent depths or conjugate depths.





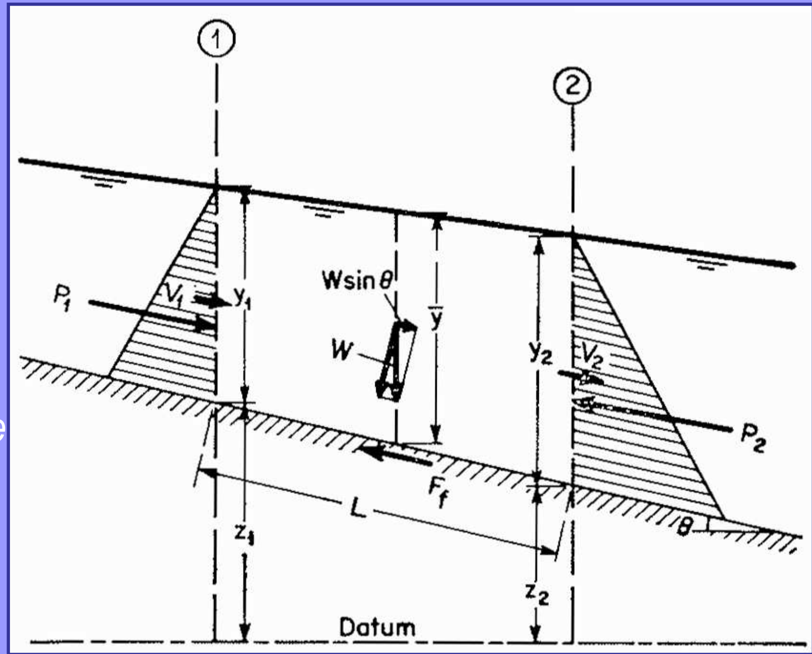


OPEN CHANNEL FLOW: hydraulic jump

OPEN CHANNEL FLOW: hydraulic jump

Due to the loss of linearity and to the unknown energy loss we have to revert to the Momentum balance

$$\beta\frac{\gamma Q^2}{gA_1}+\Pi_1+W\sin\theta=\beta\frac{\gamma Q^2}{gA_2}+\Pi_2+T_f$$



where

Π: pressure force acting on the given section, computed as

$\Pi=\gamma y_g A$ where y_g is the depth of the centroid of flow area A

W: weight of the water enclosed between the sections;

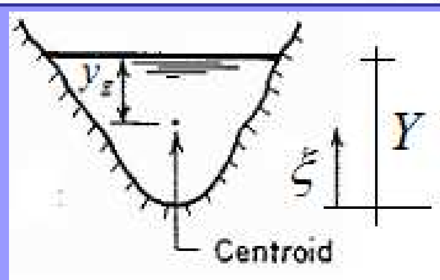
T_f: total external force of friction acting along the boundary.

If the tractive force on the boundary and the component of the weight compensate each other, we can write

$$\boxed{S=\frac{\gamma Q^2}{gA_1}+\gamma y_{g1}A_1=\frac{\gamma Q^2}{gA_2}+\gamma y_{g2}A_2}$$

According to which Specific Force S in conserved across the hydraulic jump.

This function has some interesting properties. For instance

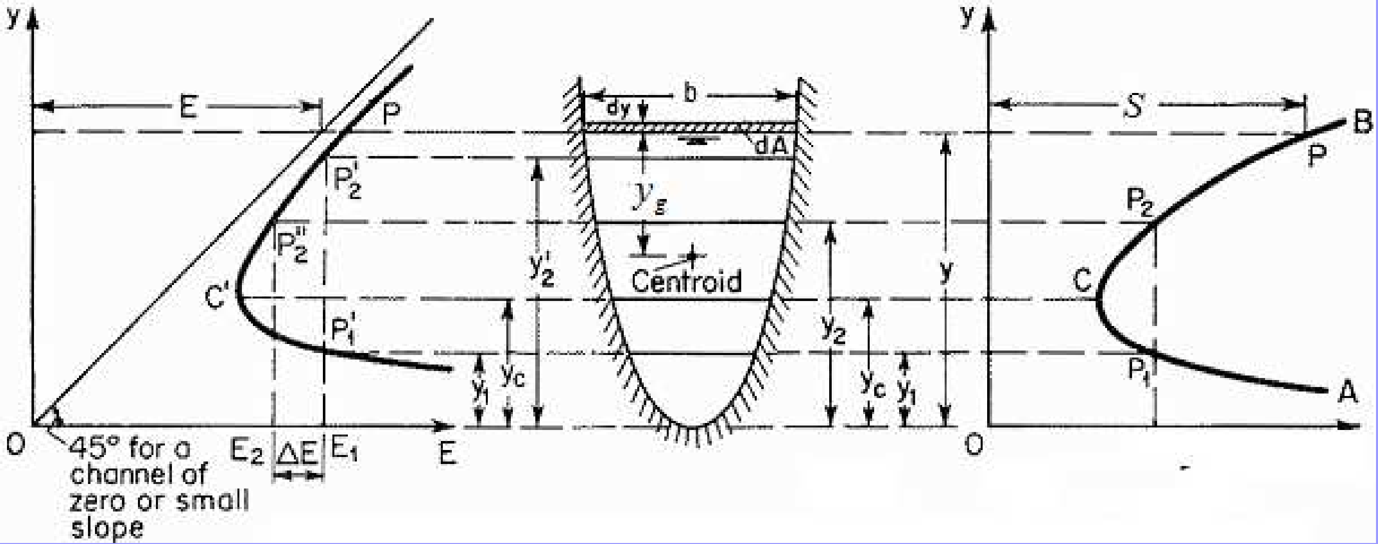


$$\boxed{\frac{\partial S}{\partial y}=-\frac{Q^2}{gA^2}\frac{\partial A}{\partial y}+\frac{\partial(y_g A)}{\partial y}=-\frac{Q^2 b}{gA^2}+A=A\left(1-\frac{Q^2/A^2}{g\,A/b}\right)=A\left(1-Fr^2\right)}$$

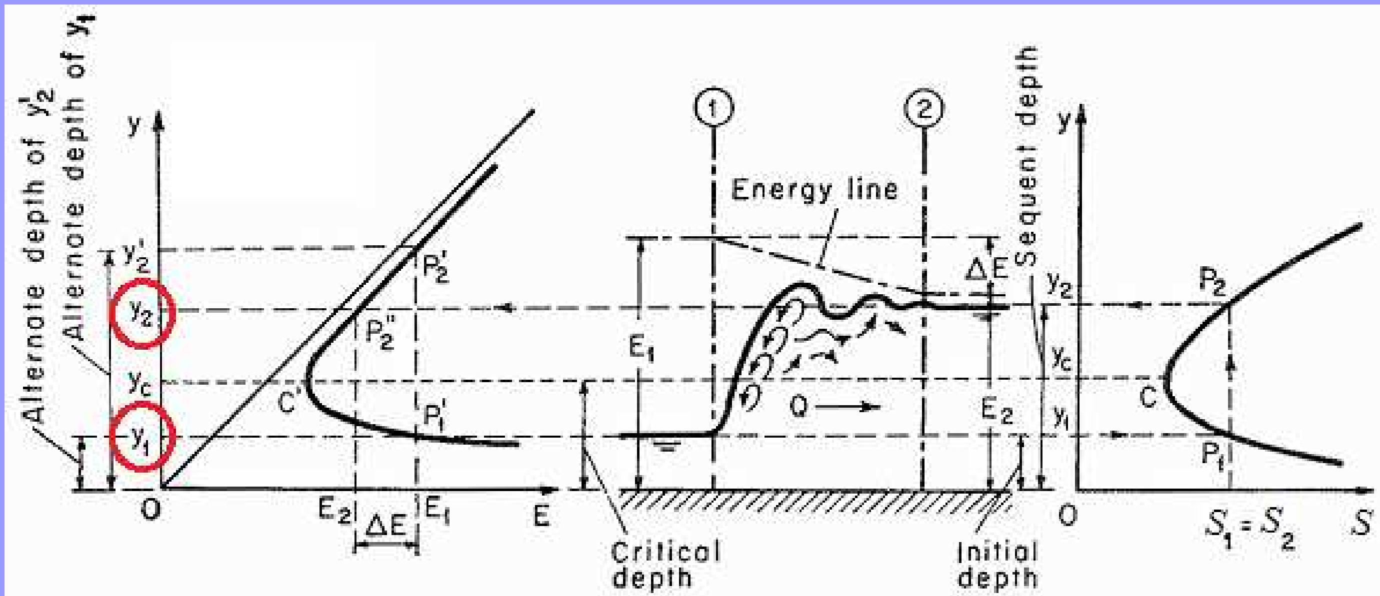
$$y_g=\frac{\int_0^Y (Y-\xi)b(\xi)d\xi}{A};\quad \frac{d(Ay_g)}{dY}=\frac{d}{dY}\int_0^Y (Y-\xi)b(\xi)d\xi=\frac{d}{dY}\int_0^Y f(Y,\xi)d\xi=f(Y,Y)1-f(Y,0)0+\int_0^Y\frac{d}{dY}f(Y,\xi)d\xi=\int_0^Y b(\xi)d\xi=A$$

Is zero when Fr=1, I.e., in critical conditions

OPEN CHANNEL FLOW: hydraulic jump



From the momentum equation in the form S_1=S_2 it turn out that since E_2<E_1 an energy loss ΔE=E_2-E_1 takes place across the jump



y_1 is the initial depth (depth before the jump) and y_2 the sequent depth.
Both are coniugate depths

For rectangular sections the condition of momentum conservation between sections 1 and 2 can be written as

$$\frac{Q^2}{gby_1}+b\frac{y_1^2}{2}=\frac{Q^2}{gby_2}+b\frac{y_2^2}{2} \quad \Rightarrow \quad y_1y_2(y_1+y_2)=\frac{2Q^2}{gb^2}$$

Whose solution, due to the symmetry of the equation, can be put in one of the followig forms that can be used to calculate downstream (or upstream) depth once upstream (or downstream) conditions are known:

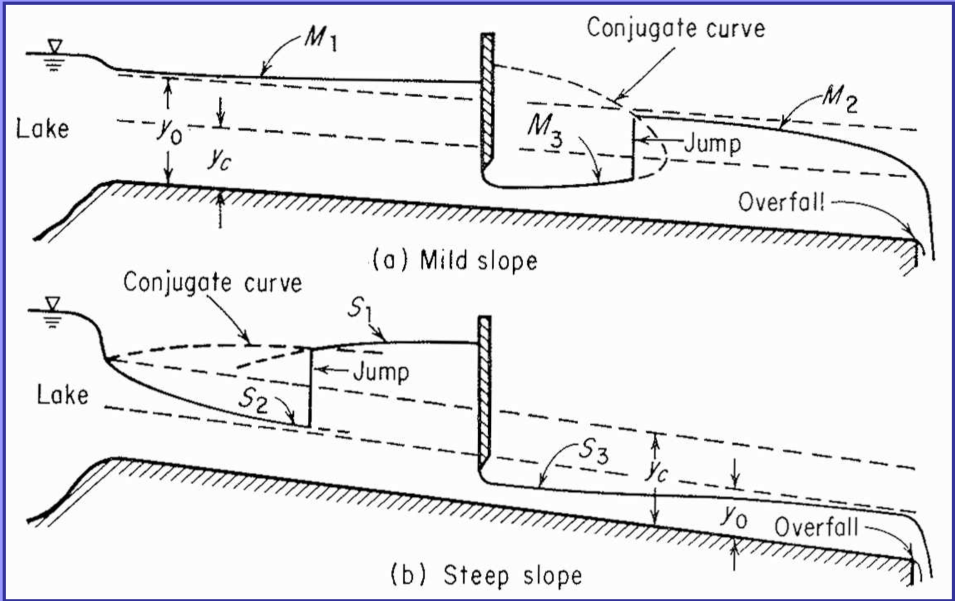
$$\frac{y_2}{y_1}=\frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right) \qquad \frac{y_1}{y_2}=\frac{1}{2}\left(\sqrt{1+8Fr_2^2}-1\right)$$

The energy loss across the jump can be calculated as:

$$E_1-E_2=y_1+\frac{Q^2}{2gb^2y_1^2}-y_2-\frac{Q^2}{2gb^2y_2^2}=\frac{(y_2-y_1)^3}{4y_1y_2}$$

Experimental investigations show that in the range ?<Fr1<? the length of the jump is L ≅ 6y2??

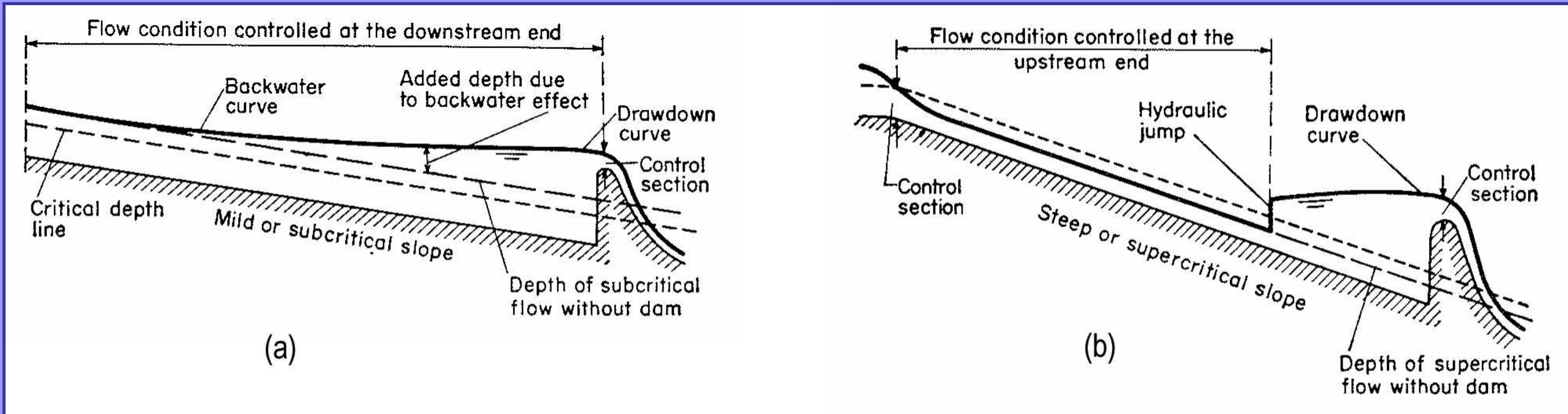
OPEN CHANNEL FLOW: qualitative profiles in complex channels



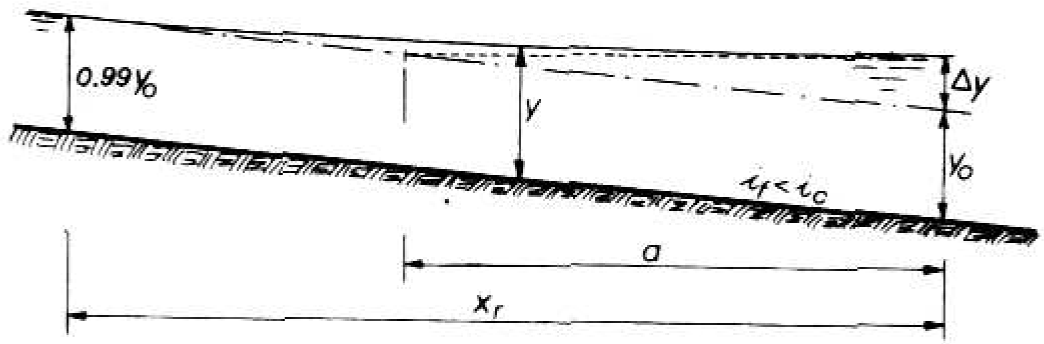
Real cases can be obtained by combining the simple profiles seen before.

As a first step control section must be identified, where the depth is known as a function of Q. From there one starts computing the profile moving in the direction dictated by the Froude number, as far as the critical depth is reached.

At this stage, in some stretch of the channel, more than a single profile is potentially present. The final choice will be the one whose Specific Force prevails.



Initially, in order to understand what is the actual applicability scope of uniform motion we asked how much far away is far ? When an M1 profile is considered a first guess can be provided by the following table, that is valid for infinitely wide rectangular channel, according to Bresse's solution



$\frac{y_0 + \Delta y}{y_0}$	8,3	6,0	4,3	3,5	2,7	1,9	1,5	1,3	1,25
x_r/a	1,05	1,1	1,2	1,3	1,5	2,0	3,0	4,0	5,0

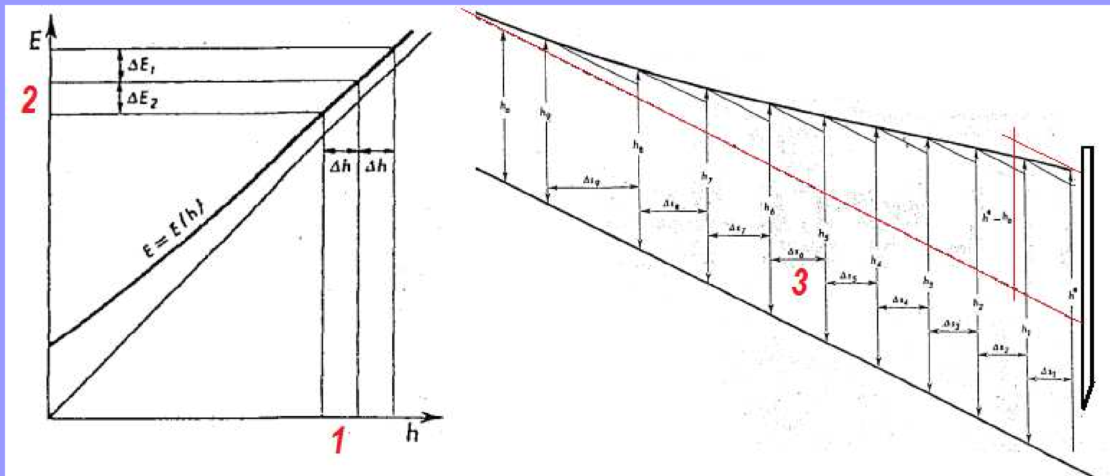
OPEN CHANNEL FLOW: quantitative profiles in complex channels

Let us first consider a method which is very convenient but is valid <u>only in prismatic channel</u>, where, indeipendently from x, one knows A(x) and $S_b(x)$

$$\frac{dE}{dx} = S_b - S_f$$

y_1 at the position x_1 along the channel is known as a boundary condition. From the qualitative discussion of the profile, one knows what is the asymptotic depth (e.g., if M_1, it will tend to h_0). Accordingly one selects in an adaptive way a depth value $y_1 < y_{i+1} < h_0$ for the section at the unknown station x_{i+1}, computing the corresponding values E_{i+1} and $(S_f)_{i+1}$ (that depend only on y_{i+1} in prismatic channels). The unknown station x_{i+1} is obtained by discretization of the dynamic equation

$$E_{i+1} - E_i = S_0(x_{i+1} - x_i) - \frac{1}{2}\left[(S_f)_i + (S_f)_{i+1}\right](x_{i+1} - x_i) \Rightarrow x_{i+1} = x_i + \frac{E_{i+1} - E_i}{S_0 - \frac{1}{2}\left[(S_f)_i + (S_f)_{i+1}\right]}$$



Direct step method (distance calculated from depth)

Let us now consider a general method which can be as convenient as the direct step if solved explicitly or just a bit more complex if solved implicitly. Its scope is not limited to prismatic channels.

We know the boundary condition y_i at the position x_i along the channel. By a first order approximation of the energy balance equation, we obtain :

$$\frac{dH}{dx} = -S_f \quad \Rightarrow \quad H_{i+1} - H_i = -\frac{1}{2}\left[(S_f)_i + (S_f)_{i+1}\right](x_{i+1} - x_i)$$

Standard step method (depth calculated from distance)

The unknown value y_{i+1} at the position x_{i+1} is such that F=0

$$\boxed{F(y_{i+1}) = 0, \quad \text{where} \quad F(y_{i+1}) = H_{i+1} - H_i + [(S_f)_i + (S_f)_{i+1}](x_{i+1} - x_i)/2}$$

IMPLICIT

This equation is non linear and must be solved by, e.g., a Newton Raphson method.

$$\boxed{y_{i+1} = y^{*}{}_{i+1} - F(y^{*}{}_{i+1}) \big/ F'(y^{*}{}_{i+1})}$$

*As a first guess for the iteration, a first estimate y*i+1 of yi+1 can be obtained as*

$$y_{i+1} = -z_{i+1} - \alpha \frac{Q^2}{2gA^2_{i+1}(y_i)} + H_i - (S_f)_i \, (x_{i+1} - x_i)$$

EXPLICIT

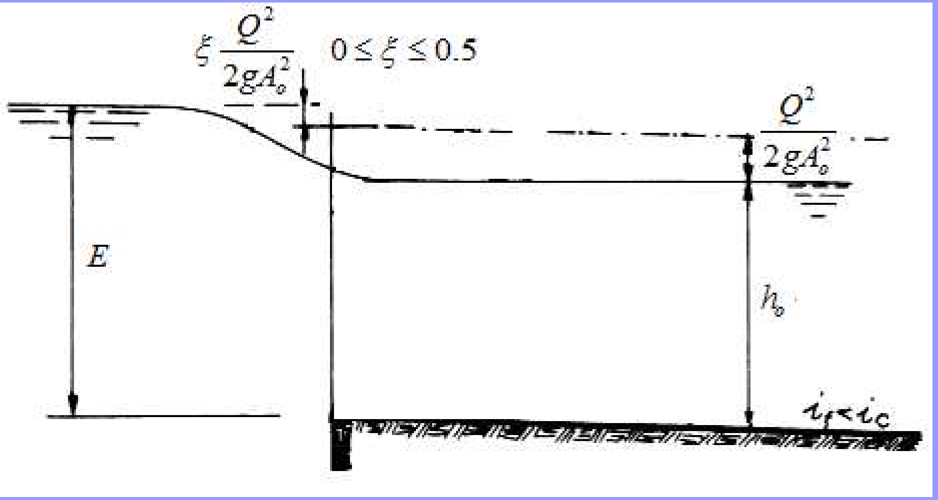
that can also be used as an explicit approximation of the energy balance equation

OPEN CHANNEL FLOW: inlet of a an infinitely long wide channel

Let us consider a wide channel (Y/B << 1) which is originated from a reservoir where water is motionless
Let us suppose that the channel is infinitely long so that we can disregard the influence of boundary conditions.
In general term we can write an energy balance between the reservoir and the flow at the inlet of the channel, also considering the presence of a local dissipation that is proportional to the kinetic energy. Q is unknown
If the slope of the channel is mild, then we should have normal depth up to the channel inlet, so that we have to solve

$$\begin{cases} E - \xi \dfrac{Q^2}{2gA_o^2} = h_o + \dfrac{Q^2}{2gA_o^2} \\ Q = \chi A_o \sqrt{R_o i} \end{cases}$$

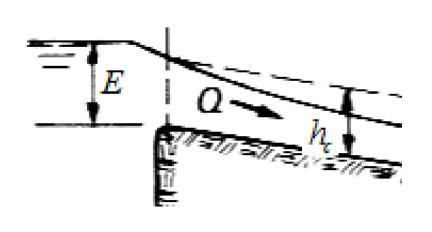
Which is solved for the normal depth and Q



Accordingly, by increasing i Q increases as well until the critical condition is obtained at the inlet

If the channel is steep, then the channel inlet is a transition through the critical depth between mild and steep channel. The system is solved for the critical depth and Q, that is indipendent from i

$$\begin{cases} E - \xi \dfrac{Q^2}{2gA_c^2} = h_c + \dfrac{Q^2}{2gA_c^2} \\ 1 = \dfrac{Q^2}{gA_c^3} \dfrac{dA}{dh} \end{cases}$$



OPEN CHANNEL FLOW: inlet of a short and wide channel - mild slope case

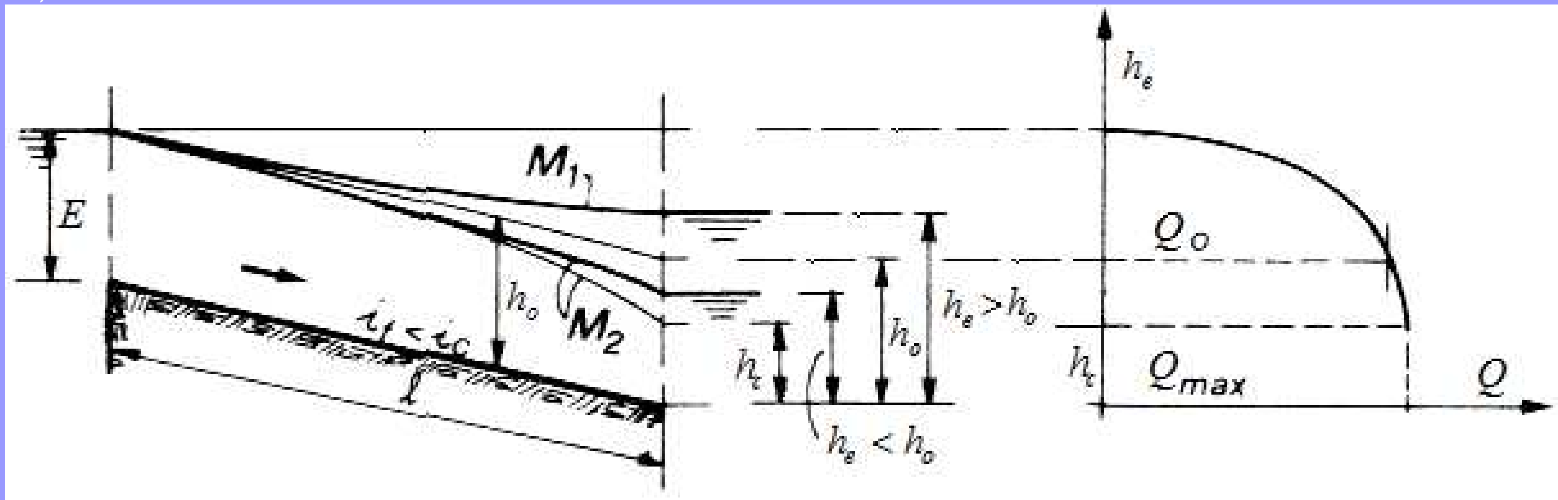
If the channel is not infinitely long, then we may have a backwater (rigurgito) or drawdown (chiamata) effect caused by the boundary condition located at the channel outlet. In this case the discharge is computed through an iterative procedure.

Let us suppose that the channel outlet is into another reservoir, that conditions the level of water at the channel end, h_e If the channel is mild, the discharge computed from the system seen before is only an initial guess

1) From Q, compute the critical depth h_c

2) If $h_e < h_c$ compute a M2 profile starting from h_c, otherwise either a M2 profile ($h_c < h_e < h_o$) or an M1 one ($h_e > h_o$)

3) Compare the computed water depth at the channel sill (inlet) with the normal depth. If it is higher, than Q must be decreased; otherwise it must be increased.

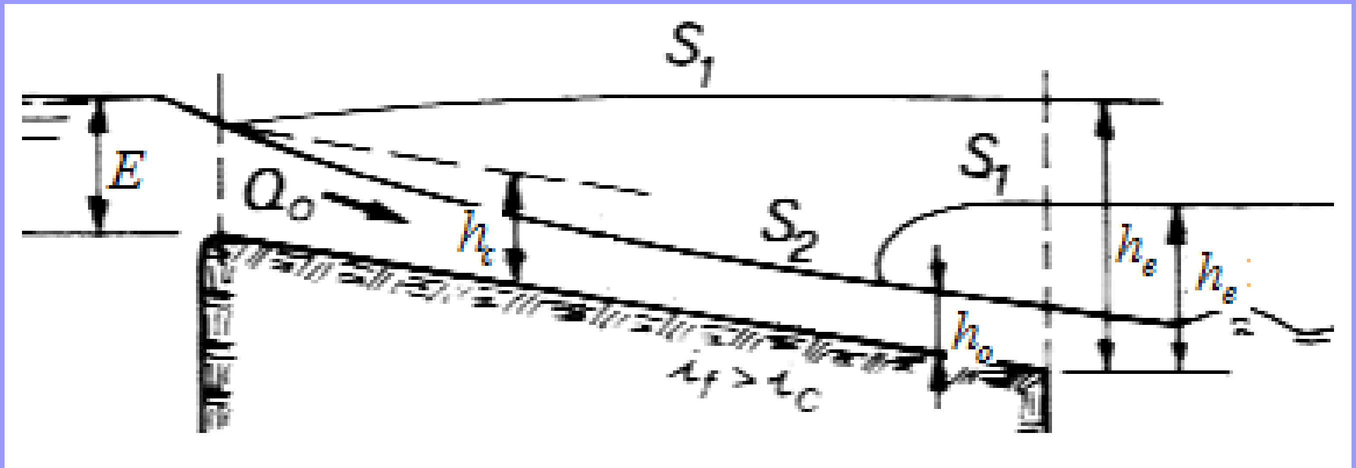


If $h_e = E + il$, then $Q = 0$;

If $h_e > E + il$, the flow is reversed from downstream to upstream. The mild slope channel turns into an adverse slope one

OPEN CHANNEL FLOW: inlet of a short and wide channel - steep slope case

If the channel is steep, there might be a backwater effect caused by the boundary condition located at the channel outlet, with an hydraulic jump that is positioned within the channel stretch. If the specific force of the S1 profile is larger than the specific force of the accelerated supercritical S2 profile, the hydraulic jump moves backward locating closer and closer to the channel inlet where eventually there might be a drowned hydraulic jump.
This happens when h_e is close to the upstream energy level E



If h_e = E +il, then Q = 0;
If h_e > E +il, the flow is reversed from downstream to upstream. The original channel turns into an adverse slope one

Now you know what "Infinitely Long" means

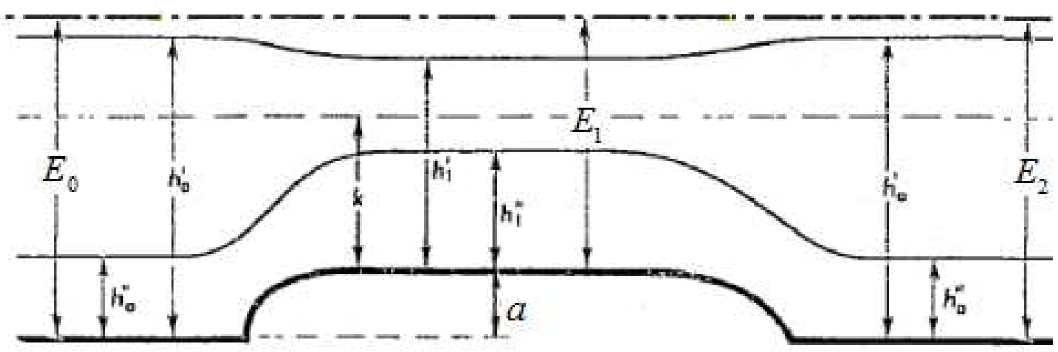
OPEN CHANNEL FLOW: passage over a sill (hump, bump: soglia)

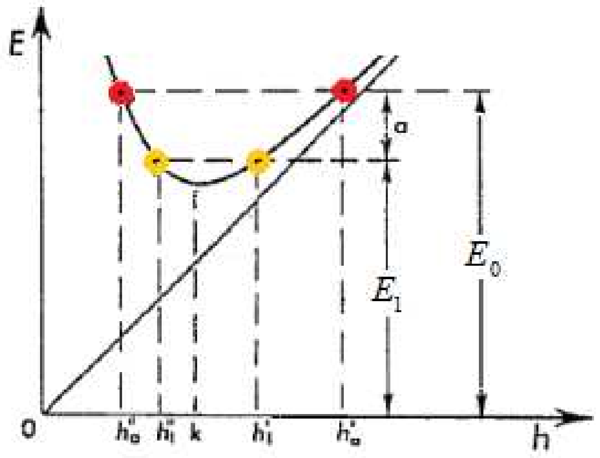
When flow passes over an hump, several situations may happen, depending on the Froude number and on Energy content. Locally there is a sudden curvature of the flow, the channel is not prismatic and the theory on water surface profiles is of no use. However an energy balance can be accomplished to study this transition

let us first suppose that no head loss is present

$$E_1 + a = E_0; \quad E_2 = E_0$$

and that the sill height a is small with respect to the energy upstream

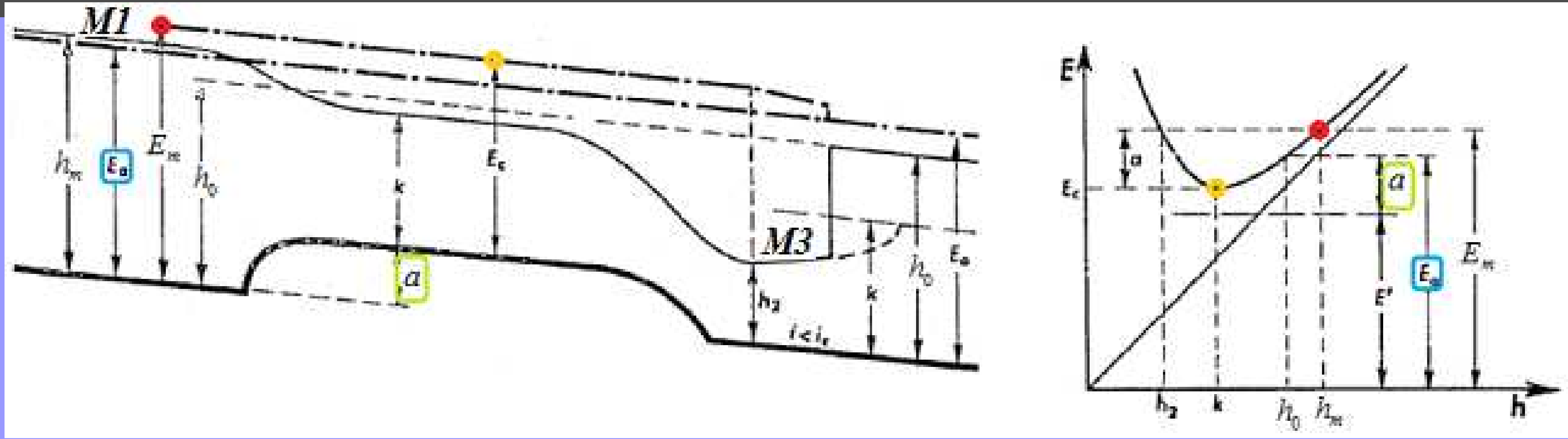


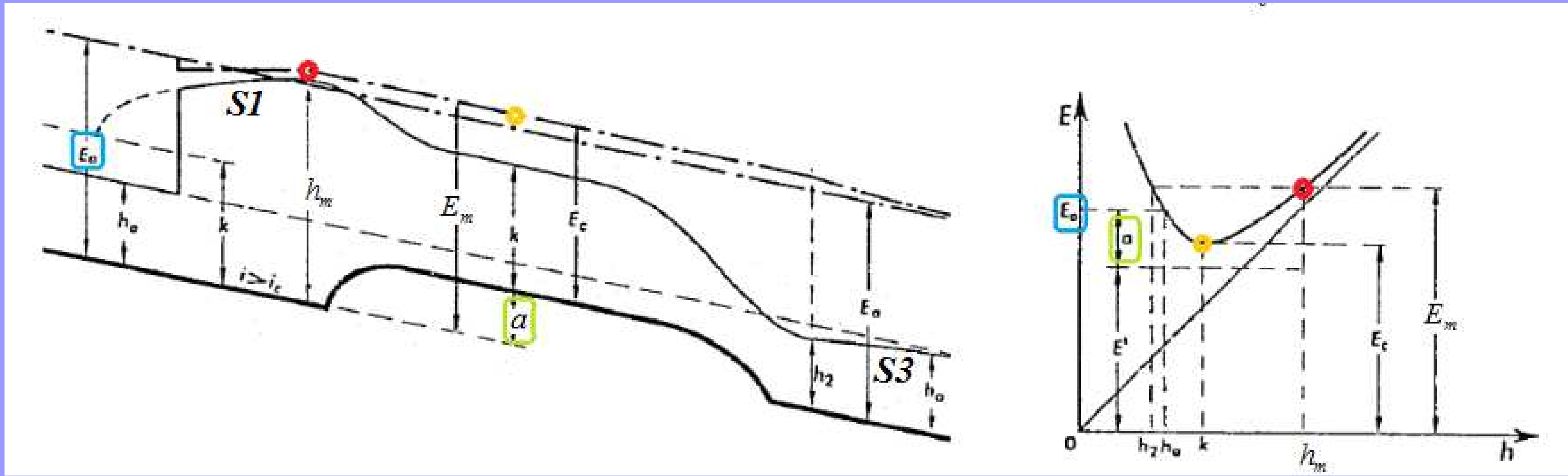


If the slope is mild, water depth on the sill lowers more than the sill height. If the slope is steep, the effect of rise of the sill bed prevails

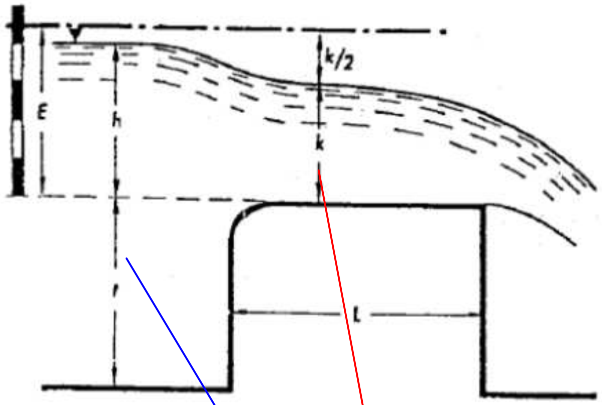
OPEN CHANNEL FLOW: passage over a sill (hump, bump: soglia)

Sometimes the height of the sill is such that the specific energy in normal flow is not sufficient to pass over it. Again, we have to distinguish between mild and steep channel





AN IMPORTANT EXAMPLE: Broad crested, round nose, horizontal crest weir

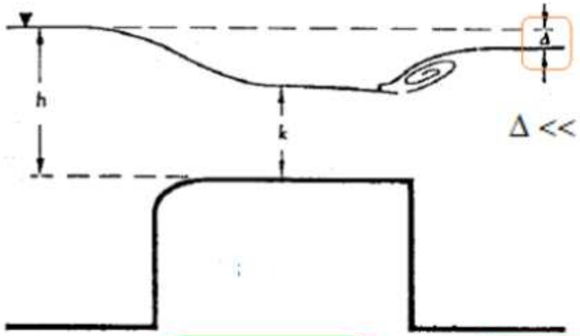


$$E = h + \alpha\frac{U^2}{2g} = \boxed{h + \alpha\frac{Q^2}{2gB^2(h+p)^2}} = \frac{3}{2}k$$

$$Q = Bk\sqrt{gk}$$

$$h + \alpha\frac{\left(Bk\sqrt{gk}\right)^2}{2gB^2(h+p)^2} = \frac{3}{2}k$$

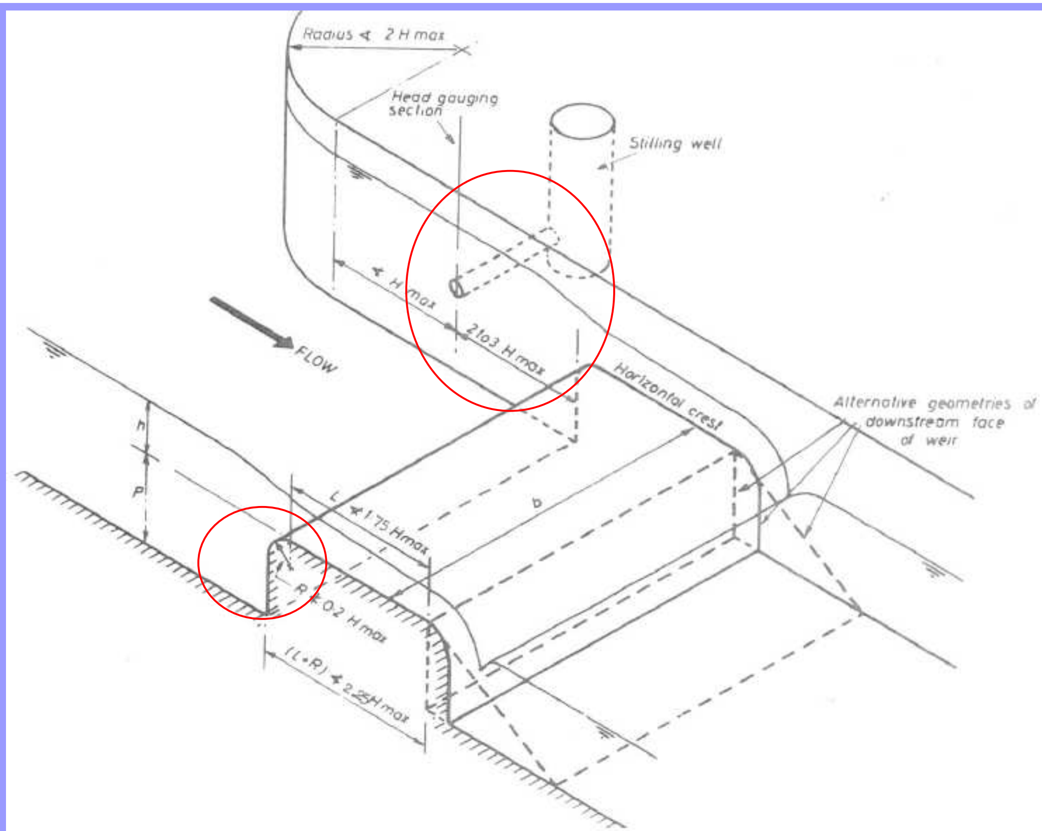
when $\frac{U^2}{2g} << h \rightarrow k = \frac{2}{3}h$; $\boxed{Q = \frac{2}{3\sqrt{3}}B\sqrt{2g}h^{3/2} = 0.385B\sqrt{2g}h^{3/2}}$



$\Delta << h$

or more precisely $\boxed{0.385C_V C_D B\sqrt{2g}h^{3/2}}$ where $\boxed{C_D = \left(1-\frac{2xL}{b}\right)\left(1-\frac{xL}{h}\right)^{3/2} < 1}$

$x = \begin{cases} 0.003 & \textit{laboratory weir} \\ 0.005 & \textit{field installation} \end{cases}$



- *Upstream corner well rounded to prevent separation*
- *Geometrical requirements as in figure above and in the specific publications*

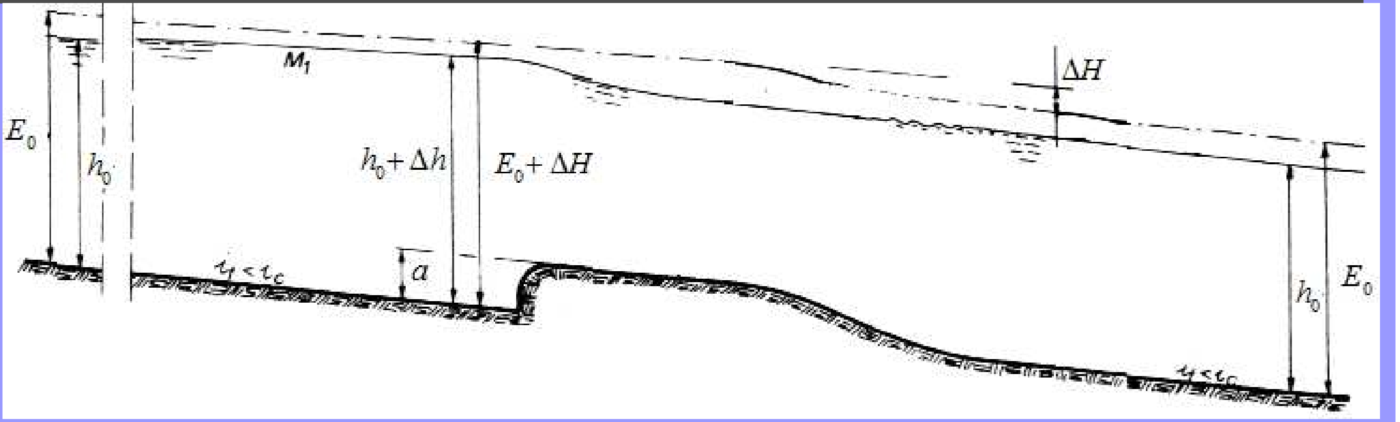
WEIRS: Broad crested horizontal crest weir

OPEN CHANNEL FLOW: passage over a sill (hump, bump: soglia)

But an head loss is almost inevitable so that (1: on the sill; 2: downstream; 0m: upstream) Making an energy balance starting downstream (2), one Sees that in a mild channel the level upstream is higher (M1)

$$H_2 \equiv H_0; \quad H_2 = H_1 - \Delta H_2; \quad H_1 = H_{0m} - \Delta H_1; \quad H_{0m} = H_0 + \Delta H$$

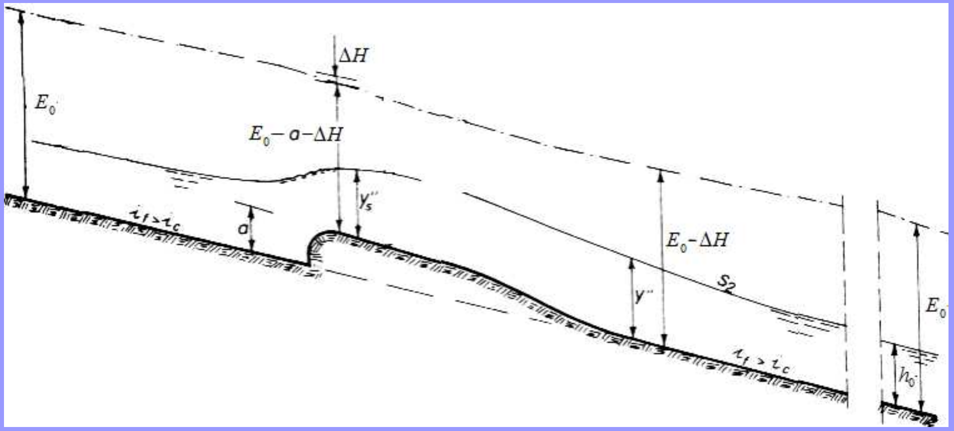
$$E_{0m} = E_0 + \Delta H$$



$$H_{0m} \equiv H_0; \quad H_1 = H_{0m} - \Delta H_1; \quad H_2 = H_1 - \Delta H_2; \quad H_2 = H_0 - \Delta H$$

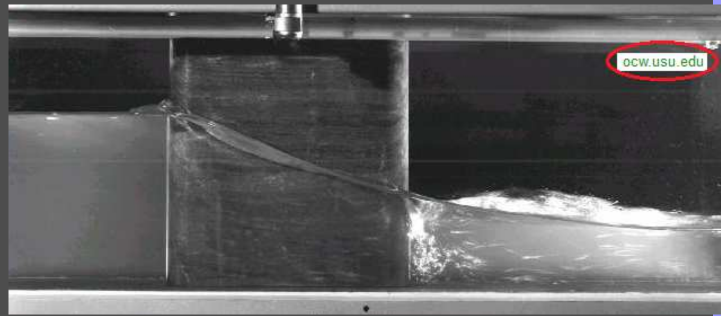
$$E_2 = E_0 - \Delta H$$

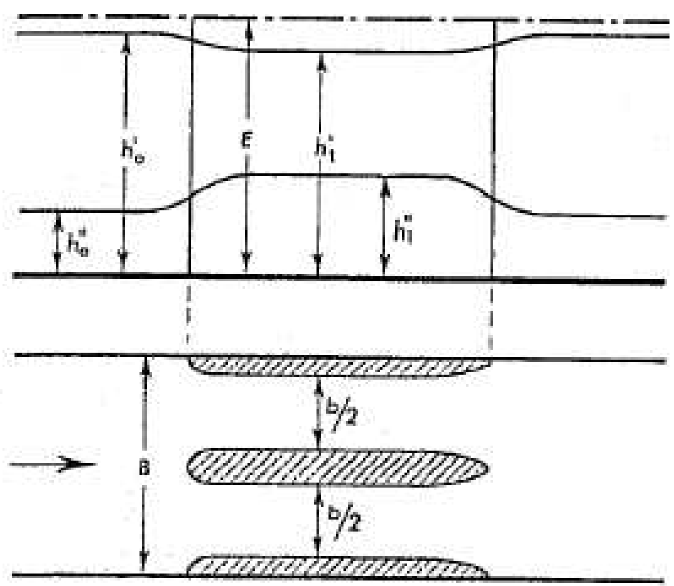
And in a steep channel, Starting upstream, one sees that the rise on the hump is stronger and The level downstream Is greater than the Normal depth (S2)

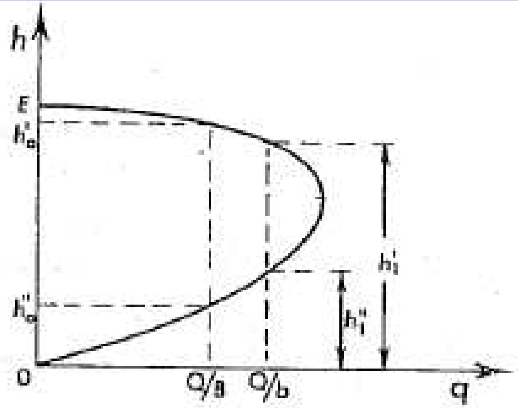


OPEN CHANNEL FLOW: passage through a contraction (1)

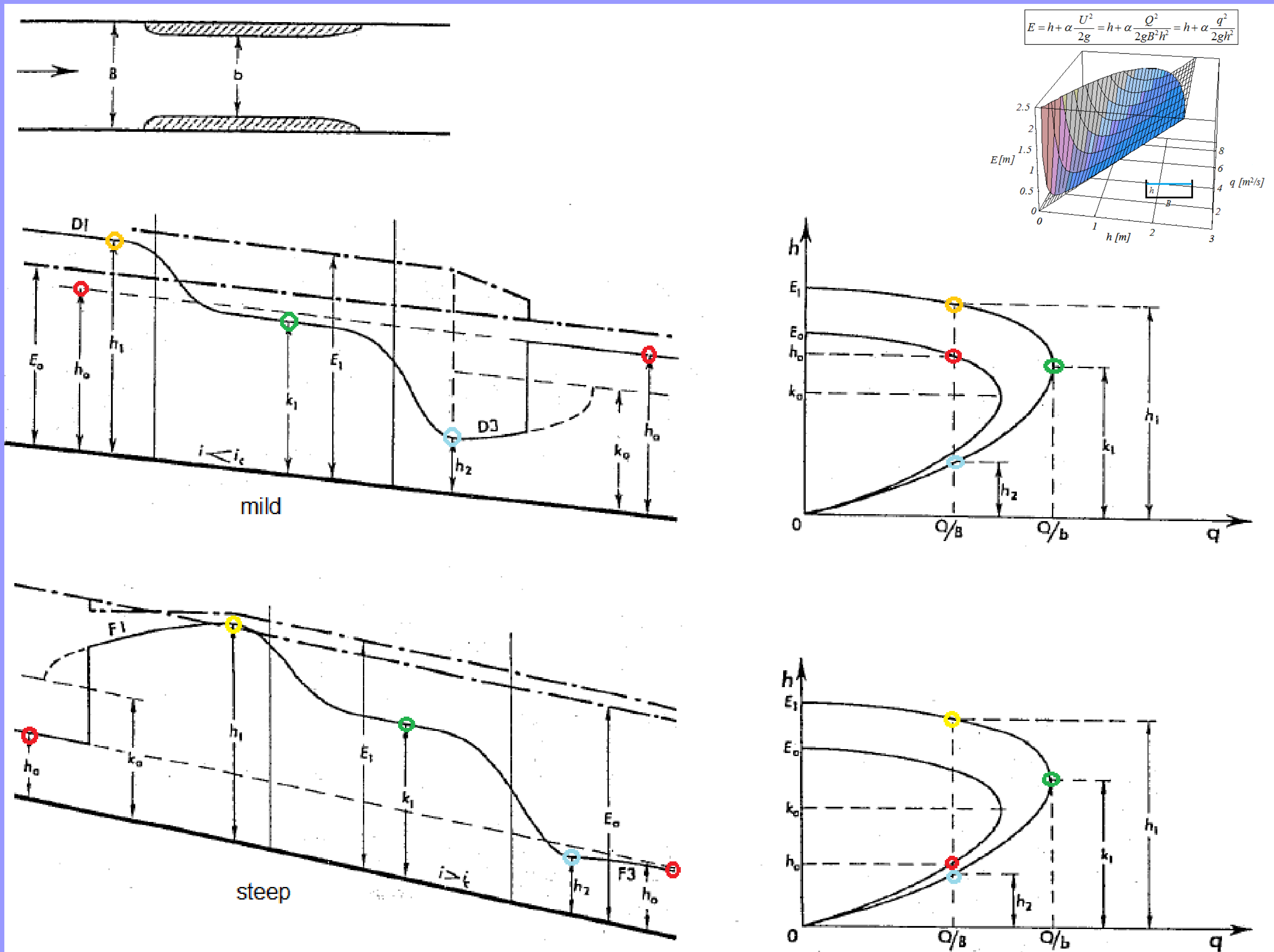
The same situation occurring when a flow passes over an hump can be observed in the passage through a contraction. Usually a contraction can be caused by the piers or abutments of a bridge





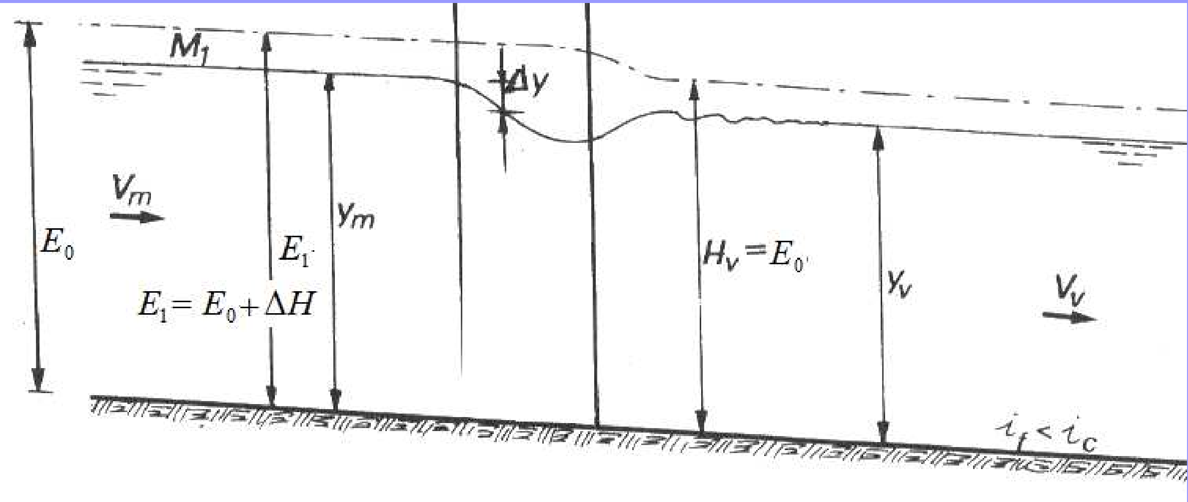


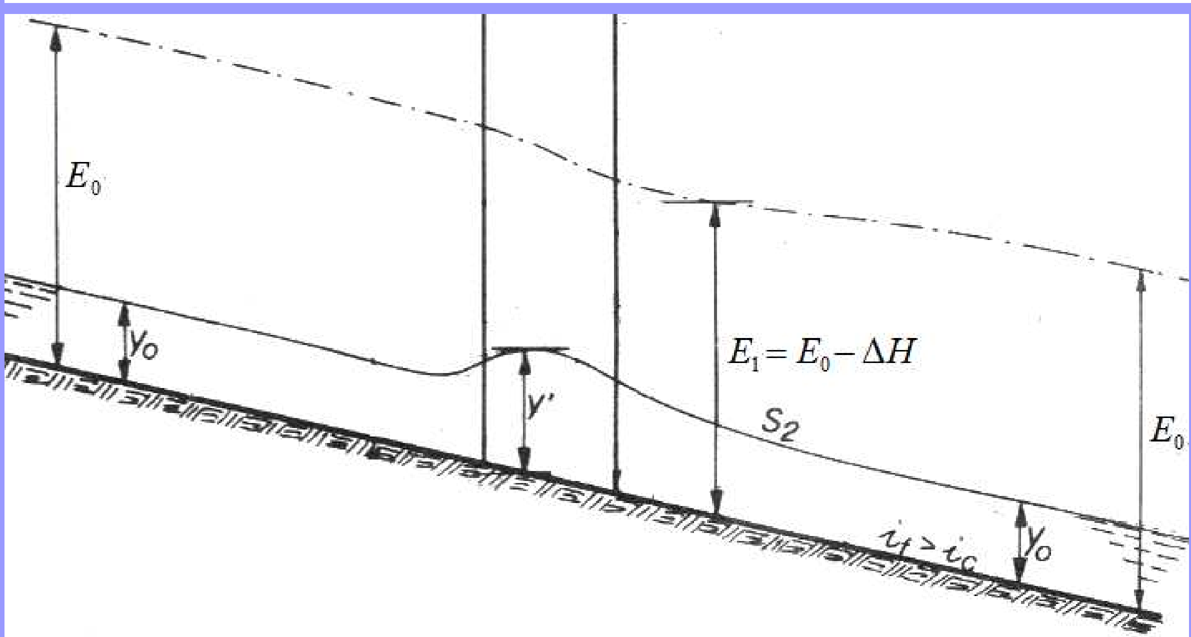
OPEN CHANNEL FLOW: passage through a contraction (2)



Sometimes the Energy upstream isn't enough…

OPEN CHANNEL FLOW: passage through a contraction (1)





Although one can suppose that no head loss is present,this is not generally true.

Accordingly, the flow must gain energy to compensate for the localized head loss. This happens upstream if Fr < 1 and downstream if Fr > 1

$$H_{0m} - \Delta H = H_V$$
$$if\ Fr < 1\ \ H_V \equiv H_0 \rightarrow M1$$
$$if\ Fr > 1\ \ H_{0m} \equiv H_0\ \rightarrow S2$$

The process is similar to the one considered for the passage over a bump

Let us consider an abrupt drop in the channel floor. If we have an head loss we cannot directly use an energy balance and we have to revert to a momentum balance, under the same assumptions usually used to derive Borda's head loss in a pipe.

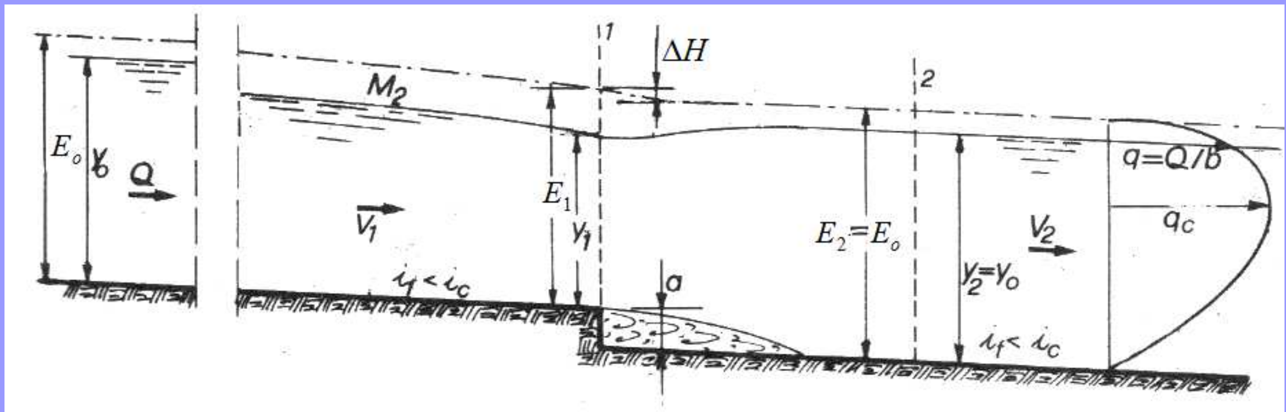
$$\beta\frac{\gamma Q^2}{gA_1}+\Pi_1=\beta\frac{\gamma Q^2}{gA_2}+\Pi_2$$

$$\frac{\gamma Q^2}{gb(h_1+a)}+\gamma\frac{b(h_1+a)^2}{2}=\frac{\gamma Q^2}{gbh_2}+\gamma\frac{bh_2^{\ 2}}{2}$$

If we now consider an energy balance

$$H_1=H_2+\Delta H$$

$$E_1+a=E_2+\Delta H; \qquad h_1+\frac{Q^2}{2gA_1^2}+a=h_2+\frac{Q^2}{2gA_2^2}+\Delta H$$

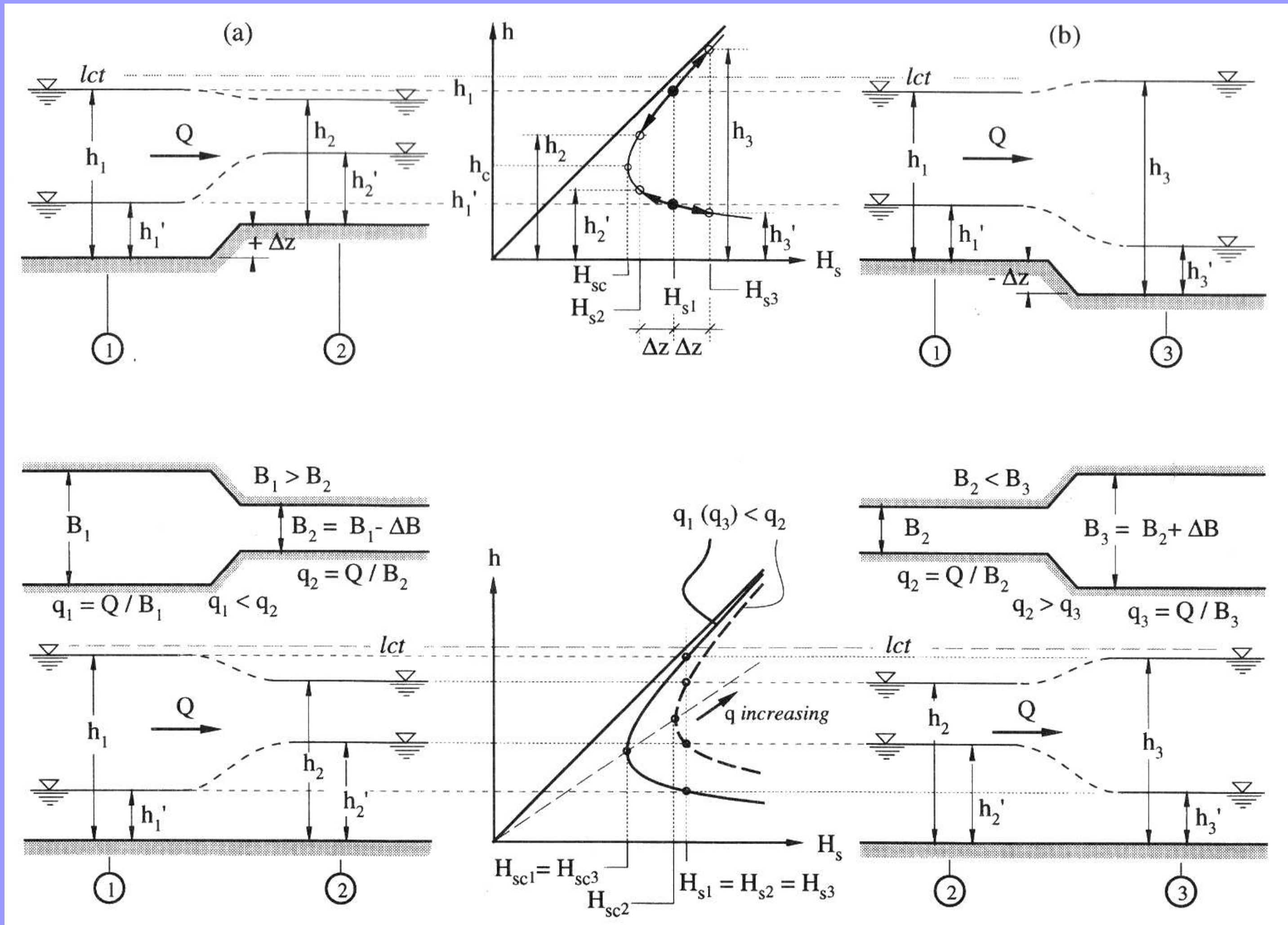


we get under reasonable assumptions (e.g., Ghetti, pag 395)

$$\boxed{\Delta H=\frac{(U_1-U_2)^2}{2g}}$$

OPEN CHANNEL FLOW: Transitions

As a first approximation one can disregard the energy losses implied in a transition. In such a case the following situations arise for a sudden rise/fall of the bed or contraction/expansion

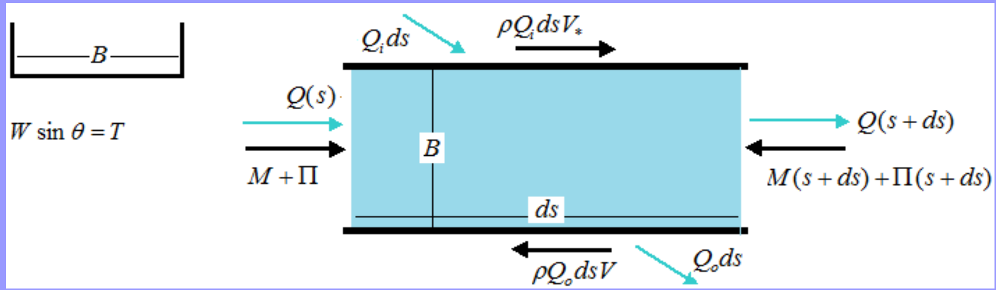


OPEN CHANNEL FLOW: Variable discharge due to lateral inflow/outflow

Main hypothesis:

- *permanent motion in a rectangular channel (base is B) with a small anc constant slope; gradually varied flow*
- *Negligibile weight component in the direction of motion and of shear along the wall;* α *and* β *= 1*

Let us consider the equation of momentum balance



$$\frac{\partial}{\partial t}\left(\int_W \rho\vec{V}dW\right) - \int_S \rho\vec{V}(\vec{V}\cdot\vec{n})dS = \int_W \rho\vec{g}dW + \int_S \vec{\sigma}_n dS$$

In its component along the main flow direction

$$M + \Pi + \rho Q_i ds V_* = M(s+ds) + \Pi(s+ds) + \rho Q_o ds V$$

$$\frac{d}{ds}(M+\Pi)ds = \rho ds(Q_i V_* - Q_o V)$$

$$\frac{d}{ds}\left(\gamma\frac{h^2 B}{2} + \frac{\rho Q^2}{Bh}\right) = \rho(Q_i V_* - Q_o V)$$

Where we suppose that the outflow velocity is V. The LHS varies with s because both h and Q are a function of s

$$\frac{dh}{ds}\left(\gamma hB - \frac{\rho Q^2}{Bh^2}\right) + \frac{2\rho Q}{Bh}\frac{dQ}{ds} = \rho(Q_i V_* - Q_o V)$$

Let us now consider the mass balance equation

$$Q(s) + Q_i ds = Q(s+ds) + Q_o ds$$

$$\frac{dQ}{ds} = (Q_i - Q_o)$$

OPEN CHANNEL FLOW: lateral outflow - Q decreasing along the flow direction

Case A: Q_i=0; discharge decreasing along the flow direction

$$\frac{dh}{ds}(\gamma hB-\frac{\rho Q^2}{Bh^2})+\frac{2\rho Q}{Bh}\frac{dQ}{ds}=-\rho Q_o V$$

$$\frac{dQ}{ds}=-Q_o$$

Which can be combined to obtain

$$\frac{dh}{ds}(\gamma hB-\frac{\rho Q^2}{Bh^2})+\frac{dQ}{ds}(\frac{2\rho Q}{Bh}-\rho V)=\frac{dh}{ds}(\gamma hB-\frac{\rho Q^2}{Bh^2})+\frac{\rho Q}{Bh}\frac{dQ}{ds}=0$$

If we now consider the flow specific energy E

$$E=h+\frac{Q^2}{2gB^2h^2}$$

It varies with s as a function of h and Q

$$\frac{dE}{ds}=\frac{\partial E}{\partial h}\frac{dh}{ds}+\frac{\partial E}{\partial Q}\frac{dQ}{ds}$$

$$\frac{\partial E}{\partial h}=1-\frac{Q^2}{gB^2h^3}$$

$$\frac{\partial E}{\partial Q}=\frac{Q}{gB^2h^2}$$

Drop Intake of a small hydropower plant

Lateral outflow (on the left) and inflow (on the right)

OPEN CHANNEL FLOW: lateral outflow - Q decreasing along the flow direction

If one consider that

$$\gamma h B \frac{\partial E}{\partial h} = \gamma h B - \frac{\rho Q^2}{B h^2}$$

$$\gamma h B \frac{\partial E}{\partial Q} = \frac{\rho Q}{B h}$$

The momentum balance equation can be written as

$$\gamma h B \frac{\partial E}{\partial h}\frac{dh}{ds} + \gamma h B \frac{\partial E}{\partial Q}\frac{dQ}{ds} = 0$$

or, more simply

$$\boxed{\frac{dE}{ds} = 0}$$

water overflow from the channel happens without decreasing the energy per unit weight of the water flowing in the channel. Its value will be determined on the basis of the boundary condition

And alternatively

$$\boxed{\frac{dh}{ds} = -\frac{1}{\left(\frac{gh^2B^2}{Q} - \frac{Q}{h}\right)}\frac{dQ}{ds} = -\frac{\sqrt{2g(\overline{E}-h)}}{gB(3h-2\overline{E})}\frac{dQ}{ds}}$$

in an alternative way, this equation provides the water surface profile differential equation. It can be integrated numerically.

Both equations require an additional equation for water overflowing out of the channel. Usually it is in the form

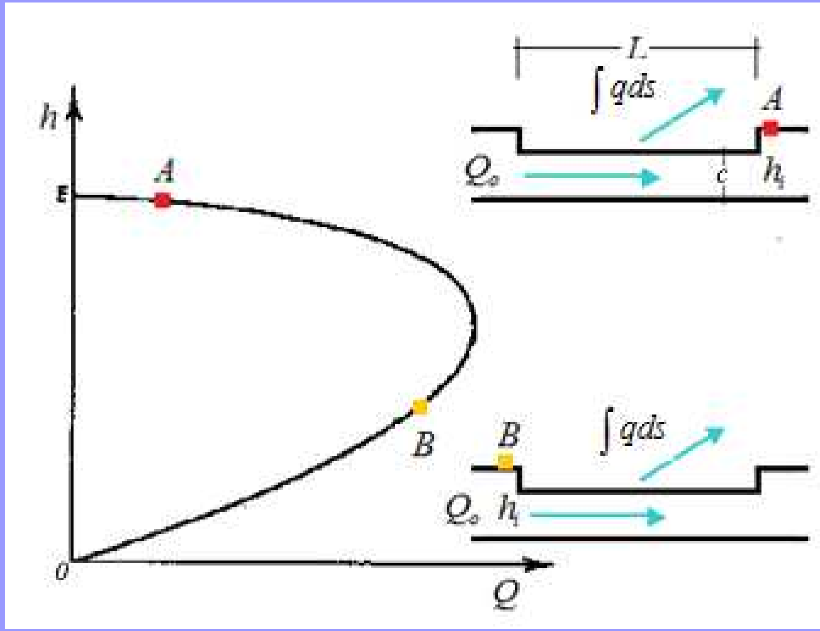
$$\frac{dQ}{ds} = -Q_o = -\mu\sqrt{2g}\,(h-c)^{3/2}$$

Although an analytical solution is possible if μ is constant, a numerical solution provides a more general approach

OPEN CHANNEL FLOW: lateral outflow - Q decreasing along the flow direction

E constant and Q decreasing along the flow: use of the Specific discharge curve

$$\frac{Q}{B} = q = h\sqrt{\frac{2g}{\alpha}(E-h)}$$



Two different classes of problem:

(1) L and c are given; find out $\int qds$ *, i.e., Q(L) - CONTROL problem*

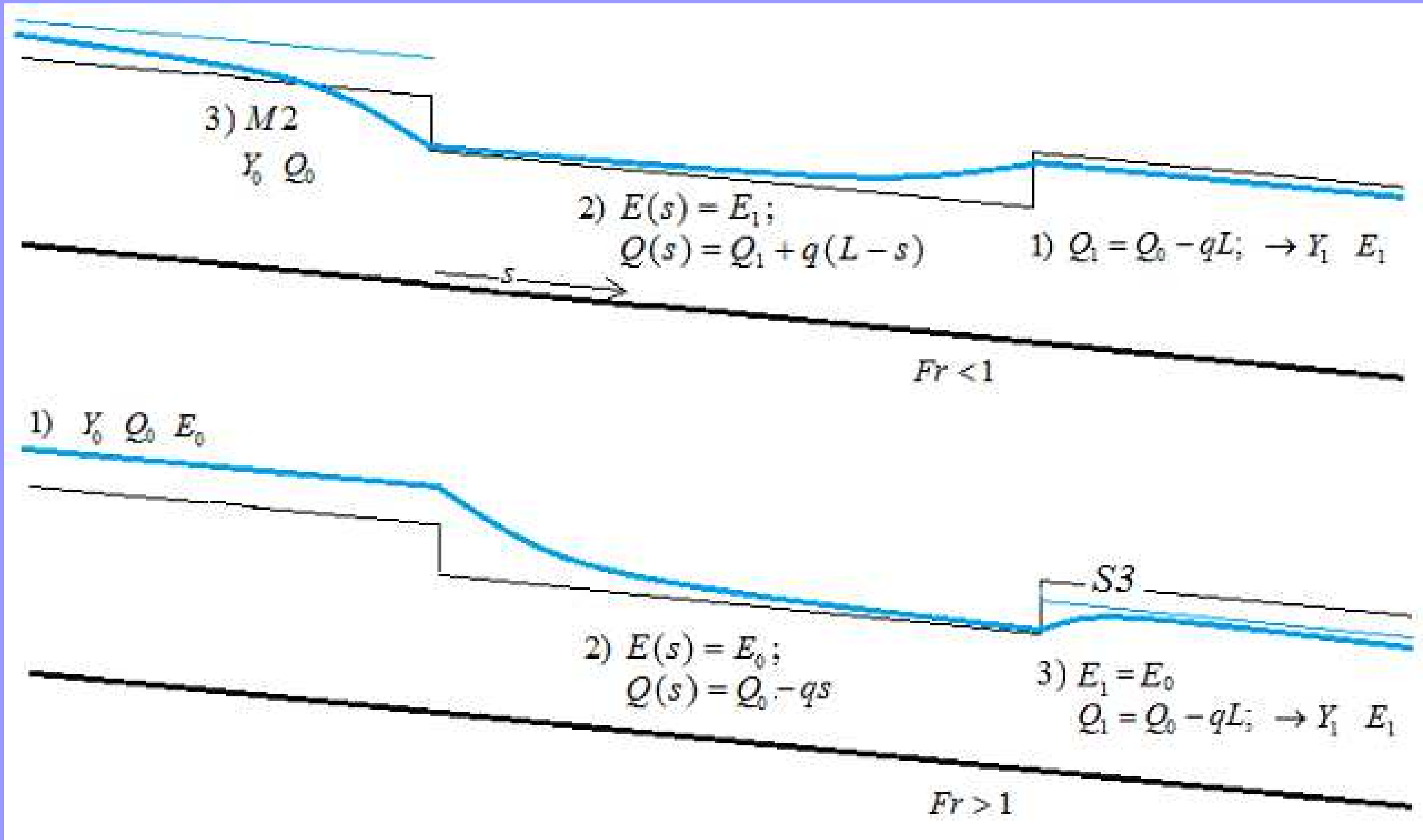
If Fr < 1, starts downstream (station A) with a temptative Q(L) and a corresponding $h_i(Q(L))$ *and compute profile in a backward fashion. Change Q(L) until Q(0) is found. Note that* $h_i(Q(L))$ *depends on the boundary condition downstream.*

If Fr > 1, starts upstream (B) knowing h_i *and Q(0) and compute Q(L).*

(2) $\int qds$ *is given; find out L or c - DESIGN problem*

If Fr < 1, starts downstream (station A) with the known value Q(L), h_i *and compute profile in a backward fashion. When Q(s)= Q(0), then L = s. If Fr > 1, starts upstream (B) with the known value Q(s),* h_i *and compute profile until Q(s)=Q(0) -* $\int qds$ *. then L = s.*

OPEN CHANNEL FLOW: lateral outflow - Q decreasing along the flow direction



OPEN CHANNEL FLOW: lateral inflow - Q increasing along the flow direction

Case B: discharge increasing along the flow direction

$$\frac{dh}{ds}\left(\gamma h B - \frac{\rho Q^2}{Bh^2}\right) + \frac{2\rho Q}{Bh}\frac{dQ}{ds} = \rho Q_i V_*$$

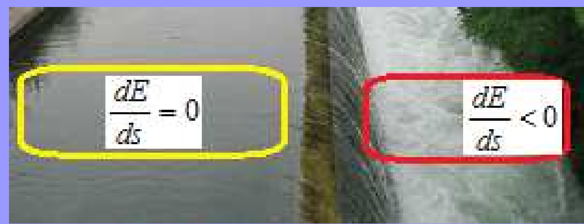
Here we need the velocity component V_* along the flow directon of the entering discharge. Often this quantity can be set = 0, so that

$$\frac{dh}{ds} = -\frac{\dfrac{2\rho Q}{Bh}}{\left(\gamma h B - \dfrac{\rho Q^2}{Bh^2}\right)}\frac{dQ}{ds}$$

Which is an equation stating the conservation of the specific force (SF)

$$\frac{d}{ds}(M + \Pi) = 0$$

Accordingly, the SF is constant whilst E certainly is not. The SF value must be determined on the basis of the boundary condition. This equation must be considered along with the mass conservation equation



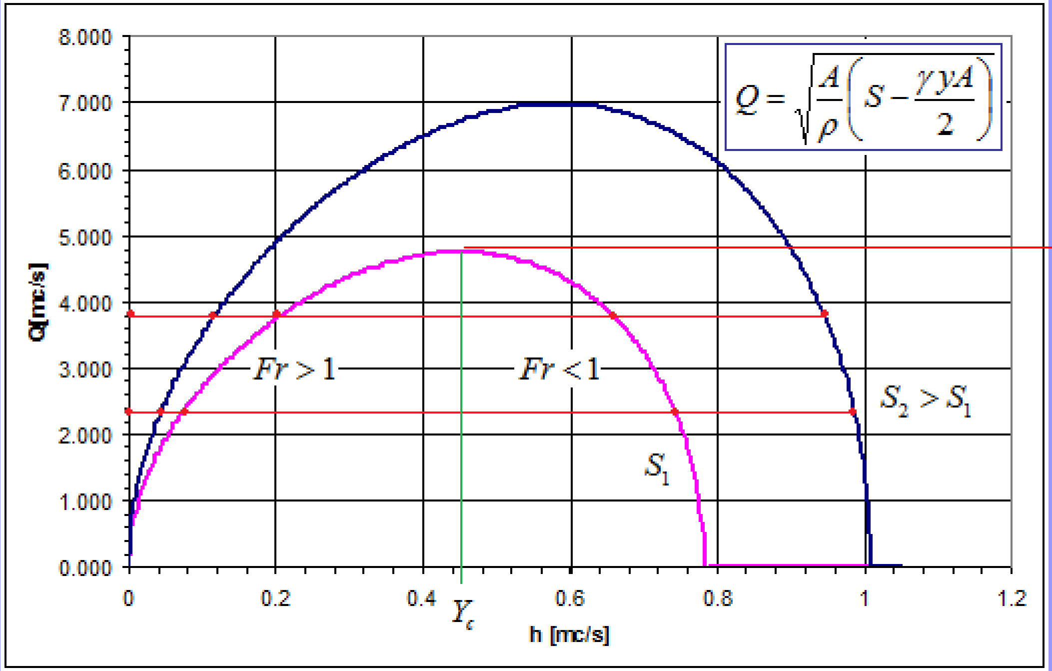
$$\frac{dQ}{ds} = Q_i$$

and where the entering discharge Q_i is a known function.

In order to investigate the possible profiles, we consider

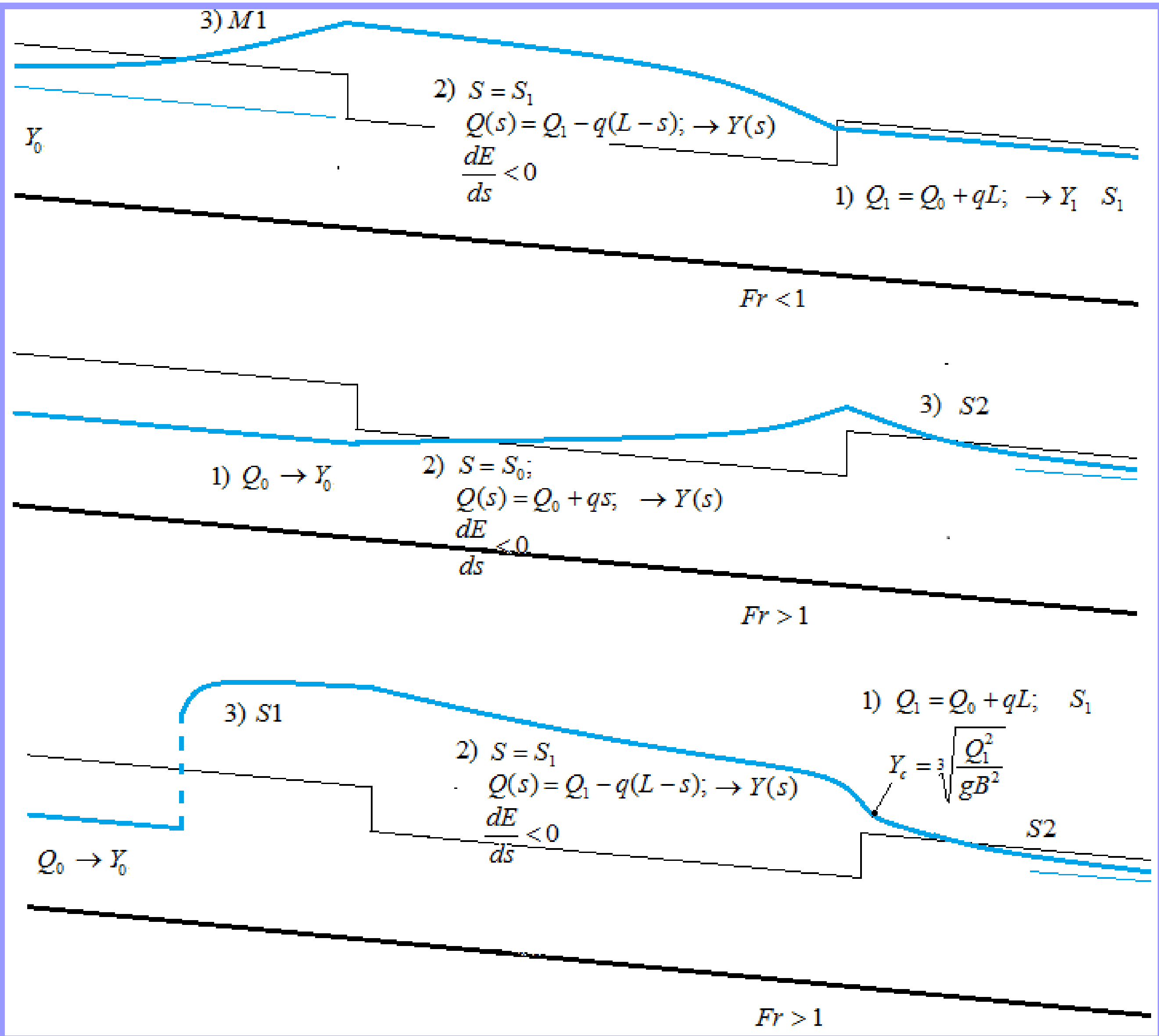
$$S=\frac{\rho Q^2}{A}+\frac{\gamma y A}{2};\quad Q=\sqrt{\frac{A}{\rho}\left(S-\frac{\gamma y A}{2}\right)}$$

whose maximum is the critical depth. As one can see, whilst Q increases with s, in a subcritical flow the depth decreases. the contrary happens in a supercritical flow. In both cases the section where the critical depth occurs can only be located downstream. In both cases, E decreases moving from upstream to downstream, due to the entering discharge that has no momentum in the average flow direction



$$Q=\left(\frac{2}{3}\frac{S_1 B^{1/3}}{\rho g^{1/3}}\right)^{3/4}$$

OPEN CHANNEL FLOW: lateral inflow - Q increasing along the flow direction



In this case, only an S2 profile is possible. Actually E, which is a specific quantity, keeps decreasing along the flow entrance flow stretch, because dq enters with 0 momentum in the flow direction. Accordingly, at the end the flow must gain energy to attain a final downstream normal flow that is more energetic the the one upstream

If Fr> 1, it might happen that the overall inflow cannot be supported by the specific force of the normal flow upstream. In such a case this situation may occur. Being a mild profile, one must start downstream from the critical depth and compute the profile moving upstream

When flows interact with the invert of a bridge, a sudden reduction of the hydraulic radius happens, so that also the stage-discharge relationship of the bridge is modified.

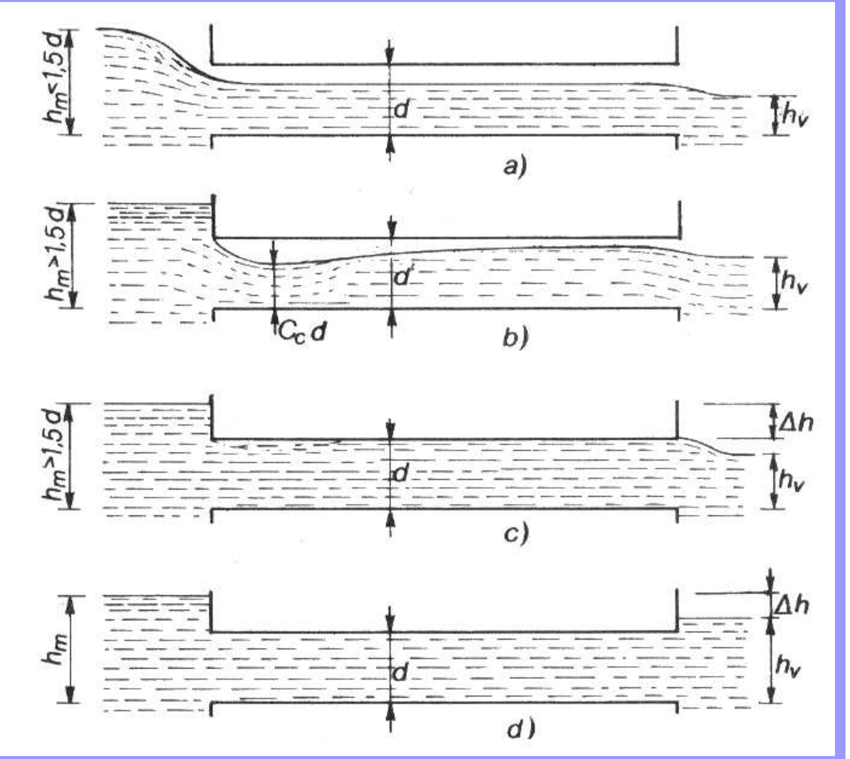
The upstream propagating M1 profile is strongly conditioned by the boundary condition exerted by the bridge

Firenze, 1966, Ponte Vecchio

OPEN CHANNEL FLOW: Culvert (tombino o botte a sifone)

Often a small channel is use to convey water from one side to the other of a levee (often a road). The hydraulic behaviour can be quite complex and, apart from the geometry, depends on the level upstream (h_m) and downstream (h_v) and on the culvert length (L).

a) Initially, when both h_m and h_v are small: open channel flow through a contraction
b) Then, when h_m grows but both L and h_v are small: orifice flow
c) Eventually, pressure flow



The transition between 1 and 3 implies a reduction of RH.
Accordingly, a strongly backwater effect may occur

OPEN CHANNEL FLOW: Bridge

Bridges are the most common obstruction and they can strongly condition the upstream water surface profile. On the other hand, a wide variety of situations is possible and this must be treated on a case by case basis

In general terms, passage through a bridge usually implies a contraction, due to piers and abutments.