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Reconstruction of Clastic Porous Media

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Abstract. In this paper we present a refinement of an algorithm (Pilotti, 1998) to generate three-dimensional granular media by deposition of spherical grains in a viscous fluid. The proposed improvements allow the construction of clastic porous media made up of irregular grains, with controlled level of angularity, sorting and porosity. On the basis of visual comparison with prototypal cross sections and of computed two points correlation functions, we argue that the intergranular void spaces resulting from this procedure provide a satisfactory reproduction of the micro-geometry of several clean consolidated sandstones and can be used to explore the effect of void topology on the flow field properties.

Key words: intergranular void space.

1. Introduction

Over the last decade there has been a growing interest in the experimental and numerical analysis of transport properties in porous media at the micro and meso-scale. The logical conviction that the understanding of basic physical processes can enhance our level of comprehension of transport processes at higher spatial scales, firmly substantiates this approach. In the same way, it has long been recognised that a great deal of the complexity of the flow processes in porous media (e.g. the histeresys between drainage and imbibition capillary curves) is strongly controlled by the complexity of the underlying boundary topology. In order to explore this connection it is necessary to provide adequate representations of the three-dimensional intergranular space. While insights on void size distribution can be provided by volume-controlled mercury injection technique, the three-dimensional intergranular void space of a real sandstone could theoretically be reconstructed from consecutive serial sections taken from a plug. However, this is a delicate operation, limited by the impossibility of preparing cross sections with a spacing of less than about 10 μm (Dullien, 1991). Although this limitation can be overcome by using high resolution X-ray computer tomography for imaging the three-dimensional microstructure of a porous medium (Jasti *et al.*, 1993), an interesting alternative is provided by the computerized reconstruction of realistic intergranular geometries. This could be done either on the basis of the pore space autocorrelation function measured on two-dimensional sections (e.g., Adler *et al.*, 1992; Bentz and Martys,

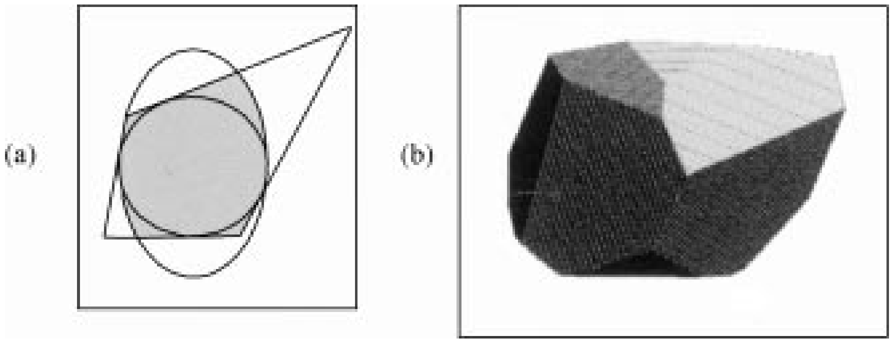

Figure 1. 2-D (a) example of X set (grey area) and 3-D clast (b), obtained with 10 randomly distributed tangency points.

1994) or by 'physically based' algorithms (e.g. Perrier *et al*., 1995; Oren *et al*., 1998; Pilotti 1998). While the former technique is conceptually hindered by the problem of pore space connectivity, crucial for porous media flow properties but questionably reproduced by an autocorrelation function, the latter reconstruct the intergranular space by simulating some relevant physical processes that lie at the basis of pore space formation. Although the scope of X-ray tomography is definitely more general in its ability to reproduce the complexity of real porous media, there are some advantages of physically based algorithms that make them worthy of investigation. In general, these algorithms have no resolution limitations and provide a tool by which many realizations of the same process can be easily created, making possible the analysis of processes which are inherently stochastic in their nature. Even more important, in the writer's opinion, they provide a numeric laboratory by which the influence of single processes on flow properties can be enucleated and investigated, in a relatively simple way, without the need of important laboratory facilities. In this context, a methodology has been recently presented (Pilotti, 1998) to generate three-dimensional granular porous media by simulating the sedimentation process of spherical grains in a still fluid. Although the constraint on the spherical shape of the grains can be removed, the types of porous media that can be reproduced by that technique are basically limited to aggregation of rounded grains. In this contribution we present an extension of the algorithm that allows the construction of porous media made up of angular, irregularly shaped particles, like the ones that are usually found in several sandstones.

2. Numerical Construction of Realistic Sandstones

The procedure described in this contribution is based on the post-processing of the main result of the algorithm proposed by Pilotti (1998), i.e. the analytical description of a porous medium through an array P of coordinates ($\mathbf{x}$, r). The P set fully describes the spheres that have been positioned by the sedimentation process and whose radius r honours a given probability distribution. Although the

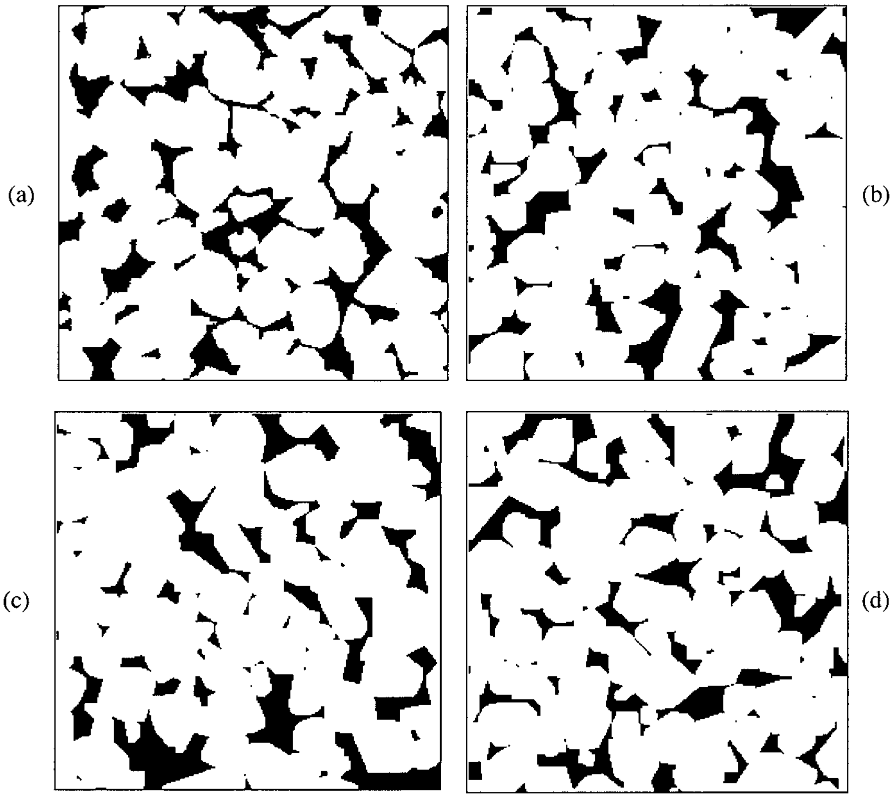

Figure 2. Binarized thin section of a Fontainebleau sandstone (a), ($\phi = 0.17$) and of reconstructed intergranular pore space (b,c,d) from a sample generated according to mixture B of Dexter and Tanner (Pilotti, 1998), where average porosity has been decreased down to 15%.

tangency condition between spheres is a straightforward consequence of the nature of the algorithm, if one observes real specimens it can be recognised that grains are often compenetrated, due to compression, cementation and cement overgrowth. Accordingly, it seems that the tangency condition can be relaxed without loss of realism.

The idea at the basis of the extension is that of inscribing each element *p* of *P* in a convex polyhedron π, whose sides are tangent to *p* in a set of points whose number and position on the surface of *p* must be aptly chosen. Each polyhedron is in turn externally limited by an ellipsoid of rotation *e*, whose centre is x_p and whose axes (a, r_p, r_p) are randomly directed. As shown in Figure 1 for a two-dimensional case, a clast is the set of points **X** such that $\mathbf{X} \in \{e \cap \pi\}$, and this operation, repeated for the whole set *P*, allows the substitution of the originally generated porous medium with a binary representation of a clastic porous medium (e.g. Figure 4).

Some further explanation is now necessary. Firstly, the number of tangency points on *p* governs the angularity of the clast, for the greater this number, the

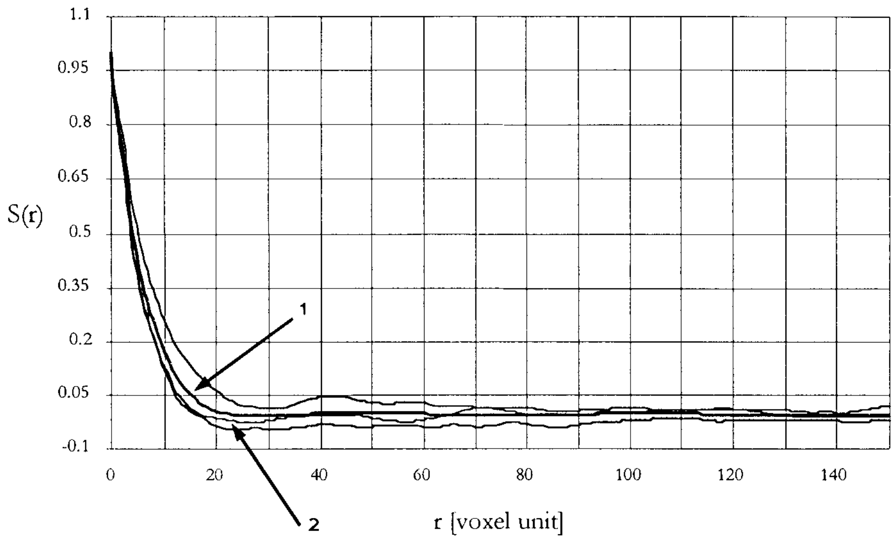

Figure 3. Average two point correlation function for the reconstructed pore space (1) and for the Fontainebleau section (20). The outer lines show the variation range of $S(r)$ observed in the reconstructed pore space.

more rounded the grain is. Their position can be generated randomly on the surface of the sphere although, if one deems it significant, it is a matter of rather simple trigonometry to choose the points so that the dihedral angles between the sides of the polyhedron correspond to those of a desired crystal. As far as the ellipsoid of rotation e is concerned, the choice of $a > r$ allows the reproduction of elongated clasts, while the two axes of length r ensure that, if one measures particle size by using a sieve diameter, the grain mixture resulting from the above outlined transformation honours the same grading curve of the set P, so preserving a sorting level close to that of the original mixture.

Since the initial porosity of the sample is close to that of the original sphere aggregate, a final step of the algorithm allows the porosity ϕ to be increased in order to match the desired value within a bound that depends on the resolution of the binary representation. This step simulates polyhedral cement overgrowth through an expansion of the surface of $\mathbf{X}_p$, one voxel wide, which is performed on each clast of the porous medium. After all the grains have been processed, the value of ϕ is computed, and, where necessary, the swelling operation is iterated again. From the computational point of view the whole procedure is quite simple. The main problem (i.e., given a point inside an ellipsoid of rotation and a convex polyhedron, to determine whether the point lies inside or outside their intersection), is a standard geometric problem that can be handled in quite effective ways.

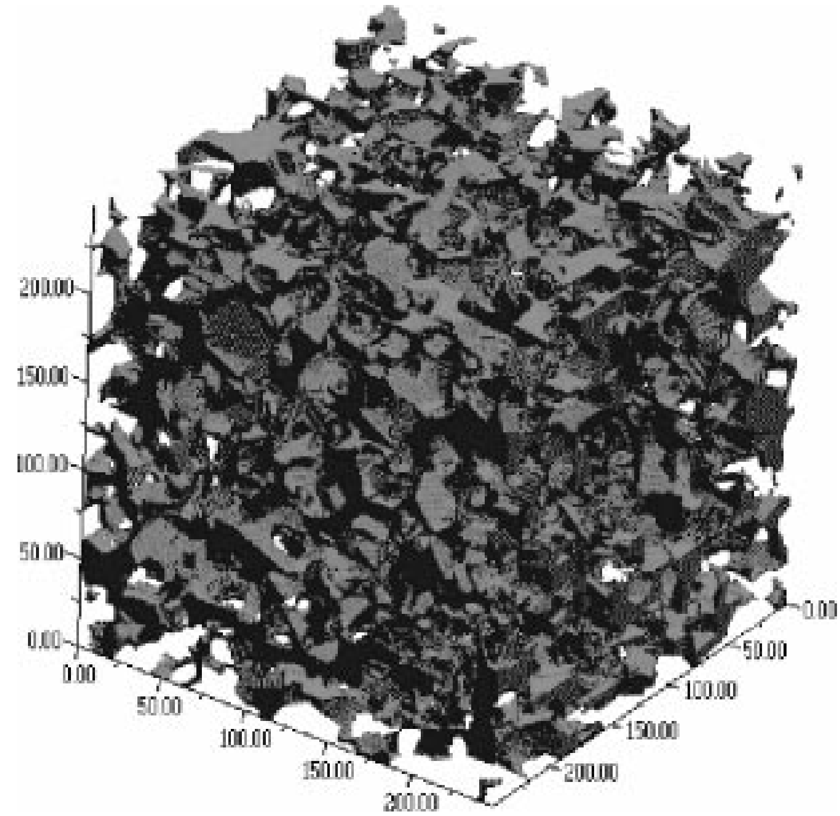

Figure 4. Example of intergranular pore space from the sample generated according to mixture B of Dexter and Tanner.

3. Comparison

The effectiveness of the methodology can be evaluated by comparing cross-sections from real specimens with reproduced ones. An example is provided in Figure 2 where we show a cross section from a Fontainebleau sandstone (a) along with cross sections (b, c, d) from the reconstructed pore space shown in Figure 4. This has been obtained by applying the outlined procedure to one of the mixtures studied by Pilotti (1998), i.e., Dexter B, which has been chosen due to its level of sorting, which is comparable to that of the prototypal specimen. Whilst the roundness has been controlled by imposing 13 tangency points on each grain, the porosity has been stepwise reduced to an average 15% which is close to the 17% of sample 2a. As a further evaluation criterion in addition to visual inspection, in Figure 3 we compare the two point correlation functions $S(r)$ computed for the real specimen and the reconstructed sample. $S(r)$ measures the probability of finding two end points of a segment r inside the same phase and has been obtained by bilinear interpolation of $S(x, y)$, defined as

$$S(x, y) = \frac{1}{\sigma M \cdot N} \sum_{i=1}^{M} \sum_{j=1}^{N} (Z(i, j) - \mathbf{Z})(Z(i + x, y + j) - \mathbf{Z}),$$

where $Z(x, y)$ is the phase function, defined as 1 if the voxel (x, y) belongs to the pore space and 0 otherwise, $\mathbf{Z}$ is its average value and σ its standard deviation. Since the reconstructed sample in Figure 4 has been digitized on a 250^3 lattice, the $S(r)$ function has been computed for 25 equally spaced cross-sections. Accordingly, in Figure 3 both the average $S(r)$ function of the reconstructed sample and the variation range for the 25 computed curves are shown. It can be observed that, as far as the correlation structure is concerned, the prototypal specimen could be regarded as a realization from the reconstructed intergranular pore space.

4. Conclusions

The proposed enhancements allow a satisfactory reproduction of clean, consolidated clastic sandstones, with a controllable level of sorting and porosity. If needed, this framework can be used as a base on which to simulate clay or cement precipitation (Oren *et al.*, 1998). The increased realism in the boundary description will make the analysis of the flow field obtained by solving Navier–Stokes equations in the intergranular pore space more informative.

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