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Monitoring the inception of sediment transport by image processing techniques

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Abstract In this paper we present a non intrusive device to detect the inception of sediment transport and to measure bed load in low-submergence flows. A solid state linear CCD camera provides linear images of the sediment movements across a control section, located on the bottom of a laboratory flume. These arrays of information are reassembled to provide a raster image of the bottom of the flume in correspondence of the control section. An on-line operating image processing software counts the number of grains and evaluates their velocity.

1 Introduction

Monitoring solid particles transported by a steady and uniform fluid flow is an important problem in hydraulic engineering. Time evolution of rivers, the design of stable channels, estimation of sediment load transported by sheetflow and rillflow down to the basin channel network are just a few examples of engineering problems where a correct assessment of sediment transport plays a key role in the design and management process. Essential to the study of sediment transport, is the prediction of the threshold of sediment transport, i.e. the average shearing stress at which a bed of incoherent particles is first considered to be in motion. In spite of its outstanding importance and of the experimental and theoretical efforts of many researchers, this point still remains an open problem in hydraulic engineering.

Although it has been demonstrated that this threshold must be a function of only two adimensional groups (Shields 1936; Yalin 1977), from the analytical standpoint the problem remains intractable, due to the scarce knowledge of the complex hydrodynamics of boundary layers and to the intrinsic stochastic nature of the phenomenon. Accordingly, the relationships available in literature for the critical stage of a movable bed are mostly empirical, expressed in terms of the critical dimensions shear stress Y as a function of the dimensionless grain Reynolds number X,

$$Y=f(X)$$

or

$$\frac{u_{*cr}}{gS_gD}=f\left(\frac{u_{*cr}D}{\upsilon}\right) \qquad (1)$$

where u_{*cr} [m/s] is the critical shear velocity, g [m/s^2] the acceleration due to gravity,

$$S_s=\frac{\rho_s-\rho}{\rho}$$

the sediment specific gravity, being ρ_s and ρ [kg/m^3] the sediment and water mass density, D [m] a characteristic sediment length and υ [m^2/s] the fluid kinematic viscosity. One of the main stumbling blocks for the study of the inception of sediment transport is its definition itself. The point is made even more obscure by the misuse of this term to indicate sometimes the inception of sediment motion, i.e. the threshold for the motion of single particles.

As a matter of fact, from the experimental point of view, there is no evidence of a definite threshold between no motion and motion (e.g., Paintal 1971; Taylor and Vanoni 1972; Graf and Pazis 1977; Lavelle and Mofjeld 1987). This is because, as originally shown by Grass (1970), the beginning of sediment motion is the outcome of the interaction of two stochastic fields. The first is determined by the arrangements of grains on the erodible bed and the second by the field of the hydrodynamic forces acting on the bed. In addition, in turbulent flow conditions the latter continuously varies over time. Accordingly, even for vanishing low shear stress, the probability that a single sediment grain moves is non-zero, and the diction threshold of sediment *motion* turns out to be inadequate because such a threshold does not exist. A direct

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consequence of this consideration is that the identification of the critical stage of particle motion *en masse*, i.e. the beginning of sediment transport, is the result of highly subjective definitions, based on the observer's perception of the process. This is evident in the original work of Shields (1936), where not a threshold line, but a shaded area separates the transport and non-transport zones on the diagram named after him. Shields observed that both particle geometry and relative position are distributed properties, so that the critical shear stress cannot be precisely identified. According to Kennedy (1995), Rouse, who was the first to understand the importance of Shields' research, substituted a solid line to the shaded area, so giving the impression of the existence of a critical tractive force where transport begins suddenly as the threshold is reached. Once the conventional nature of this line was recognized, some attempts to give a quantitative criterion for its identification (e.g. Neil and Yalin 1969), have apparently passed unnoticed. This is certainly one of the causes of the scatter which generally affects experimental data on the beginning of sediment transport.

This contribution is part of a wider research on the problem of the revision of the Shields curve (1) in laminar conditions. Several experimental data have been obtained in the last 2 decades (Mantz 1977; Yalin and Karahan 1979) that contradict the hyperbolic pattern traditionally attributed to the Shields curve (1) in the range $X<1$. In order to get some additional data and to provide a physical explanation of the proposed new trend, a set of experiments for the identification of the inception of sediment transport have been devised. However, in order to get data consistent with the ones provided by former Authors, there must be consensus on the meaning to be given to the word *threshold*. In laminar motion the problem is simpler with respect to turbulent conditions, because the shear stress field is permanent in time. According to Yalin and Karahan (1979): *In the case of a laminar flow the grains of the uppermost layer are dragged* (*all together*) *in the form of a layer* (*grain carpet*), *and therefore the determination of sediment transport in laminar flows is comparatively easier*. During a long set of experiments in laminar motion we have never observed a *carpet motion*, as a generalized collapse of the upper layer. Our observations are closer to the opinion of Wang Sany-yi and Wen Shen Hsieh (1985), that *for laminar flow… as soon as flow exceeds this incipient condition*, *nearly all the sediment particles are in motion*. In order to give a more precise definition of the meaning that must be given to the word *nearly*, it could be helpful to monitor the time distribution of the releases of incoherent grains from a selected area on a mobile bed. By analyzing the statistics of the grain detachments with time, it should be possible to give a definition of the threshold for sediment transport in terms of probability, according to the stochastic nature of the process.

In this paper we propose a device which allows sediment particles, transported by an open channel flow in laboratory conditions, to be monitored as they pass through a control section so they can be counted and their velocity measured. The problem of monitoring bed load transport is similar to other technological problems, where an image of indefinite length must be acquired (e.g., to check the quality of paper, fabric or metal tapes which move underneath an optical controlling device). Often, in these cases, the use of a linear camera has proved to be highly more efficient than the use of a matrix camera. In all linear cameras pixels are located along a single line. Consequently higher space resolution and scan speed, *ceteris paribus*, can be achieved when compared with ones obtained by matrix array cameras. A linear array camera optimizes the number of data which must be acquired to reproduce the image of a target moving object. A matrix array camera would generally lead to a considerable excess of data to be processed due to the overlap between single frames. In our problem, the image with indefinite length is the time-varying eulerian view of a cross section in a laboratory flume, through which sediments are conveyed by water. In order to cope with this problem, we propose a device made up of a linear camera controlled by a PC and focused on the control section. Operating like a conventional scanner, a specific software reassembles the sequentially acquired vector of pixels into bidimensional images portraying the motion of particles on the bottom. These images are processed on line, providing quantitative information on the monitored process. As an example of application, the inception of sediment transport is discussed and a semi-quantitative criterion for its identification is suggested. At the present stage of the research the device is especially designed for monitoring bed load in shallow viscous flows.

2
The probe and the related hardware

The experimental technique we propose is based on the electronic real time processing of a linear image acquired via a solid state linear charge coupled device (CCD) camera. The probe is allocated inside a laboratory flume where a viscous fluid is recirculated in isothermal condition (see Figs. 1 and 2). The lens is focused to a control section which is perpendicular to the flow direction. The control section, that is placed just downstream of a control area where sediment are placed at the beginning of the experiment, is scanned with a given frequency and data are real time processed by a PC based control system. A specific image processing software has been implemented that recognizes and counts the solid particles transported by the fluid. Moreover, with pseudo-spherical grains, the software evaluates the particles velocity on the bed of the flume.

The hardware for the present application was derived from the Logitech *Scanman* 32™ commercial scanner. Particularly the resolution was set to the maximum value of 400 DPI on a 4″ scan line and 1-bit acquisition mode. The original lens was kept whose focus length is 108 mm. The lens was oriented towards the channel bed. The control section was lightened by two 2*W* linear LED arrays emitting a monochromatic green light collimated by a cylindrical lens. These arrays are the same kind of device currently mounted (just one) on the *Scanman* hardware. The encoder device, which allows the mechanical triggering of data acquisition in scanners, was eliminated. In its place we used the signal outcoming by a D/A converter card directly triggered by the PC (see Fig. 3). Charge packs from the 1648 photoelements of the sensor are transferred to the CCD scroll register, at the end of each integration time (see flow chart in Fig. 4). Depending upon the threshold set via the amplifier gain control, the register bits are set either to zero or to one. Data are then transferred to the RAM buffer located on the scanner card. The acquisition software checks this buffer and, as soon as a new line is present, stops acquisition

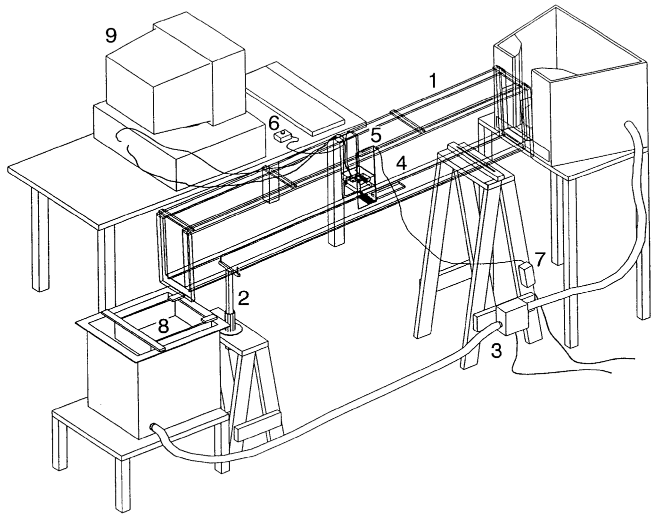

Fig. 1. Schematic drawing of the experimental apparatus. Please note the flume (*1*) with the flexible joint. Its slope is governed by a micrometer screw (*2*) and the recirculating discharge between the upstream and downstream tanks is kept by the low-head pump (*3*). During the experiments for the determination of the inception of sediment transport, sediments are placed in the patch (*4*), cut on the bottom of the flume. These sediments are then collected in the sediment trap (*8*). The measuring device (*5*) is placed in correspondence of the control section, at the end of the patch (*4*). Note the amplifier gain control (*6*), the DC power supply (*7*) and the online data acquisition and processing system (*9*) with the cables to the camera and to the AD/DA card

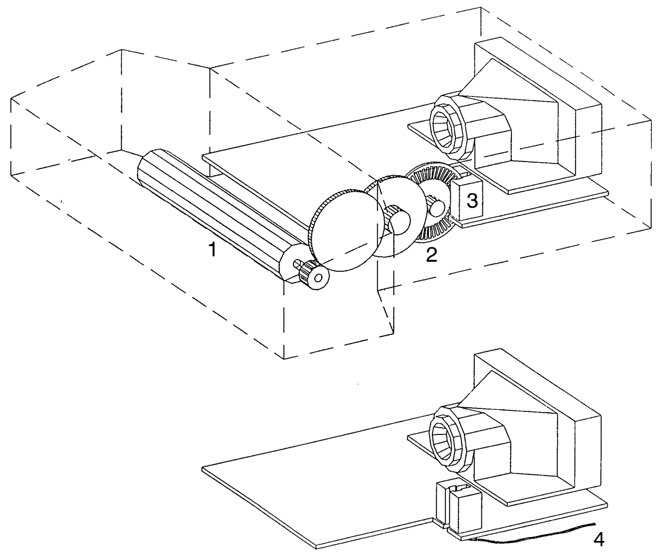

Fig. 3. Basic structure of a manual scanner. When the instrument is dragged on the image to be scanned, the contact with the surface causes the rotation of cylinder (*1*), that governs disk (*2*). This disk is partly perforated and its rotation intercepts the light emitted by the LED (*3*) of the encoder. This triggers a data acquisition impulse. In the application proposed in this contribution there cannot be any contact with the surface to be scanned (i.e., the bottom of the flume). Accordingly, the kinematic mechanism has been eliminated and the impulses are sent to the encoder by signals (*4*) coming from the AD/DA card

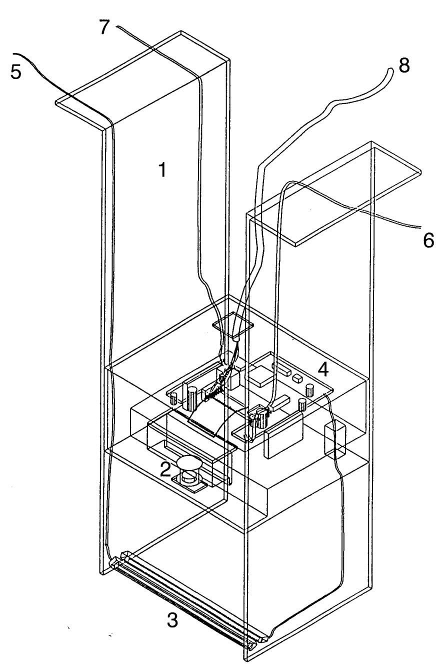

Fig. 2. The measuring device: the frame containing the instrument (*1*), the focusing lens and the sensor casing (*2*), the two LED arrays (*3*), the integrated electronics for the device clocking (*4*) and the related connections to the DC power supply to the LEDs (*5*), to the gain control trimmer (*6*) and to the cards (*7*, *8*)

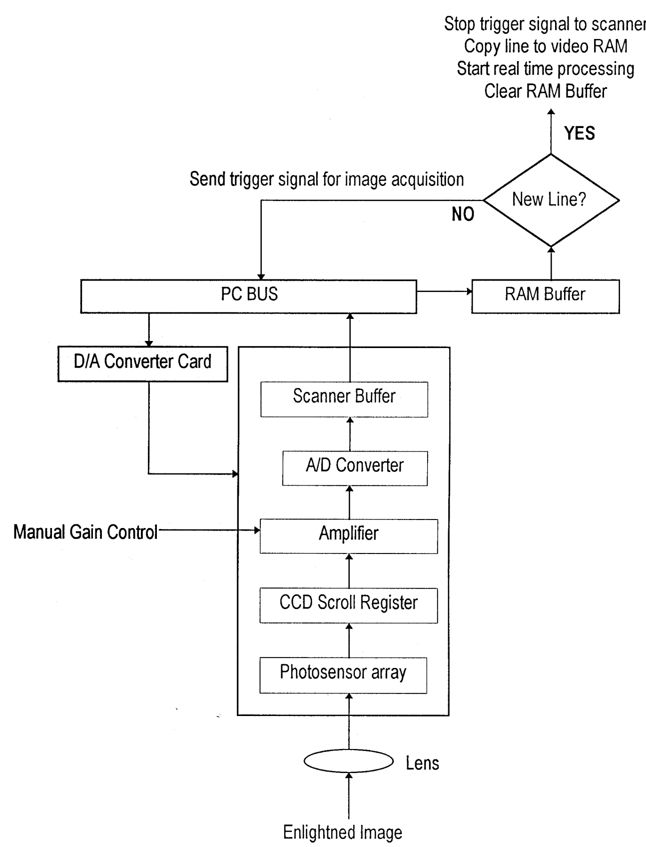

Fig. 4. Schematic flow chart of the acquisition process

triggering and when the data bus is available, copies of content to the computer RAM. A number of preliminary tests have shown that a maximum scan frequency of 370 Hz could be achieved with an Intel 486 DX based personal computer clocked at 50 MHz. The bottle neck for the line acquisition speed could be found in the time integration of the photoelements and therefore could not be increased viz. increasing the control system processor power.

Let us consider spherical particles whose diameter and velocity are D and v. If p is the pixel width (i.e. 63.5 μm in our case), and f the maximum scan frequency, the time needed for the grain to cross the control section of width p is $(D+p)/v$ and, accordingly, we can obtain the number of lines

$$n = f\frac{D+p}{v} \tag{2}$$

which are acquired by the instrument for each particle.

3 The software

The probe and its card act as a computer peripheral which is recognized by the operating system when the related driver is loaded. To this purpose a suitable command line is added in the PC configuration file where the address of the camera port, IRQ and DMA channels are specified. To access the device through its driver, a special protocol is needed which is usually supplied by the hardware producer. In our case, the SAPI (Scanner Application Program Interface) protocol was used which, via a number of C-language libraries, allows the management of the camera functions by an user written code. To this purpose, a software was developed which allows the linear images acquired by the probe to be displayed on the monitor, stored on the disk and processed in real time.

By combining the acquired linear images in a sequential fashion, a still image of the moving objects is obtained (see Fig. 5), which is generally deformed in the direction of the flow. Particularly, if the velocity of a moving grain whose linear dimension measured along the flow direction is D_p, is greater than the characteristic velocity

$$c = pf \tag{3}$$

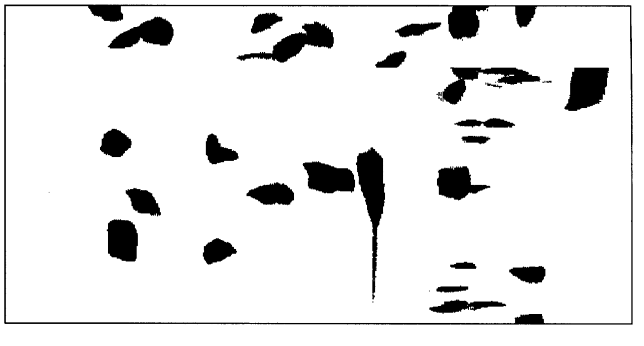

Fig. 5. Raster image showing the passage of grains beneath the linear CCD camera, across the control section. The image is obtained on-line, by reassembling the sequence of linear images sampled at 300 Hz. The irregular shape of the pseudo-spherical grains is mainly an effect of the different speed of the particles. The flow motion is from down upward

its image turns out deformed, resulting longer than the original along the flow direction, shorter otherwise. Once the amplifier gain control is properly set, a change of state occurs in the CCD pixels whenever one or more solid particles move across the control section. The image processing software detects this change of state, reassembles an image of the moving particles and recognizes the different forms so that single grains can be identified and their velocity measured. It has to be noted that single particles may slide or roll on the channel bottom; furthermore sediments often tend to cluster and move together along the channel. Accordingly, grains, even of the same size, may pass with different speeds beneath the control section. In order to cope with these difficulties, several algorithms were implemented and tested for the image processing. Their performances were first compared by simulating the probe signal via a specific computer code, generating random images of the moving grains, and then tested by physical experiences in the laboratory. Best results were obtained by the following algorithm: as a data string is read from the camera buffer it is converted into a Boolean array. If *zero* is the state pertaining to the image of the undistributed channel bottom, *nonzero* bytes in the array will be due to one or more particles passing below the lens. We call *fields* the groups in consecutive *nonzero* bytes in the array, whose length is longer than a minimum threshold, corresponding to the expected instrumental noise. Each field can be identified by its starting position in the array, and its length. Objects are detected by comparing fields acquired at each time step with those acquired at the previous one. When one or more bits show the same *nonzero* state in two subsequent time steps, the corresponding *fields* are said to *overlap*. Whenever overlapping is found between two fields, the information pertaining to the new field is attributed to the one recorded in the same position at the previous line. When no overlapping results for a given field, a new record is created, containing information regarding the identifier of the new object, the total number of pixels in the field, its maximum width and the number of lines. The number of particles is then derived by post-processing the informative content of these records. Of course, a reasonable information on the particle shape and minimum and maximum linear dimension will strongly speed up this operation and increase its reliability. This is not a particularly restrictive assumption, because during laboratory experiments for the study of bed load, the approximate shape of the grains and the grading curve of the investigated mixture are known, as part of routine preprocessing analysis.

4 Uncertainty of the measure process

Several factors can in fact affect the measure process depending on the fluid, the flow regime, the flow velocity and the sediments. As far as the viscous regime is concerned and the liquid is clear enough so that no significant suspended sediment load is present, the uncertainty factors are mainly due to the sediment color, shape, size, discharge and velocity.

Best contrast conditions between the color sediment particles and that of the control patch on the channel bottom (i.e. black on white or vice versa) where achieved to test the post-processing algorithm. Under this conditions it is easy, via static tests, to determine the correct threshold amplifier gain which allows the particles to be clearly identified on the channel bottom. When all the particles show approximately the same color, the above mentioned procedure is simplified. Otherwise, when the grains in the mixture have significantly different colors, the number of bits acquired for each pixel must be increased by one to four. In this case, as a first step to get the best quality of a digital image, a correct lightning of the control section is crucial. This has been achieved by doubling the intensity of the illumination with respect to the original hardware present in the scanner (see Fig. 2) and by searching the optimal position for the light sources. The particle shape plays also an important role in affecting the uncertainty of the measure. Actually, the system can easily determine the particle dimension D_t parallel to the CCD array, while poor information can be achieved for the dimension D_p measured along the flow direction, being this affected by the particle velocity. This, in principle, is not a problem once the sediment particles have just to be counted, but can bring to wrong results when clusters of particles close each other, passing across the control section, are to be distinguished from single, long, grains. Consequently, although spherical or sub-spherical grains will offer best measure conditions, very good results can be achieved with grains for whose the ratio between maximum and minimum linear dimension does not exceed the value of two. Again, this is not a restrictive condition, given that natural sediments are generally made up of water-worn grains, with pseudo-spherical shape.

Two different theoretical limits should be given; the first concerns the particle size D_t in the direction transversal to the main flow, that cannot be lower than the pixel size (i.e., 63.5 μm). The second limit regards the dimension D_p parallel to the flow direction, and is related to the sediment speed, v. This limit is defined by the constraint

$$v \leqslant f(D_p + p) \tag{4}$$

As can be seen from relation (2), the equal sign implies that only one scan line is acquired for a single crossing particle. Although there is not a theoretical limit to the upper value of D, practically this is dictated by the dimension of the instrument, that spans a width of 10 cm on the flume bottom and is located 4 cm from the bottom, and from the nature of the problem. In laminar conditions, such as the one investigated in the following paragraph, when the fluid is water, it can be easily shown (e.g., Breusers and Raudkivi, 1991), that D must be smaller than 1 mm. If the viscosity of the experimental fluid is higher than that of water, larger particles can be used.

In our test experiments the ratio

$$e = \frac{N_d}{N_r}$$

between the number N_d of the particles detected by the equipment and the number N_r of those released upstream was used as an index of the instrument efficiency. Experiments were performed under best contrast conditions and a target of 90% efficiency was matched in the range

$$0.5 < \frac{D_t}{D_p} \leqslant 1, \qquad 4 < \frac{D_t}{p} \leqslant 45, \qquad f\frac{D+p}{v} \geqslant 4$$

4
An experiment on the beginning of sediment transport

The instrument presented in this paper has been devised to identify the inception of sediment transport, as a part of a wider research on the identification of the Shields curve for low X values (Pilotti 1995; Menduni and Pilotti 1994). On the basis of few experimental data in the range $X < 1$, the form

$$\frac{\rho u_{*cr}}{\gamma_s D} = \frac{c}{\dfrac{u_{*cr} D}{v}} \tag{5}$$

(where c is an adimensional constant), was originally proposed for the Shields curve. Although this trend has been justified by theoretical arguments (e.g. Yalin 1977; Saffman 1965), it is highly uncertain and it has been questioned by Grass (1970), Mantz (1977), Yalin and Karahan (1979), Yalin (1992) and Pilotti (1995). In particular, on the basis of experimental and theoretical considerations, Pilotti has proposed the form

$$Y_{cr}(n) = \frac{8.33 tg(30 + 0.5n)}{\left[37.5 + \dfrac{1.499X^2}{1 + 0.4X}\right]} \tag{6}$$

for the relation (1) in viscous flow with $X < 20$, that provides the cumulative distribution of Y_{cr}, being n the percentile.

The definition of what transport is meant to be is essential to the identification of its threshold. According to Yalin and Karahan, in the hydrodynamic conditions considered in this example, the threshold can be identified without any ambiguity. During their experiments with glass beads, they observed and described the process of grains mobilization as a *carpet motion*. On the other hand, during a set of experiments with naturally worn silica grains, we observed a more gradual process of bed mobilization. Although the ambiguity in the identification of the beginning of the motion *en masse* is, in this hydrodynamic condition, considerably limited, in our opinion it is not possible to define it as a carpet motion. This is a term that conveys the idea of an instantaneous mass mobilization of the erodible bed.

The instrument presented in this paper allows a rationalization of the meaning given to the term *threshold* during our tests. The experimental apparatus is presented in Fig. 1, where the gauge with the related hardware is shown, located inside the small recirculating flume. Cut on the bottom of the flume, a small patch is visible, where sediments are placed at the beginning of the experiments. After that the sediment surface has been carefully smoothed down to the level of the non-erodible surrounding bed, the flow is started at a constant discharge Q [m^3/s], and the average shear stress exerted by the shallow flow on the erodible patch is gradually made greater by increasing the slope of the channel. This can be done by operating the micrometer precision screw placed downstream at the outlet of the flume. In this way, it is possible to pass as smoothly as possible from a non-transport situation to a fully developed one.

During the experiments, the number of particles moving across the control section as a function of time is automatically recorded on the hard disk of the PC. Along with this information, the flow condition inside the flume is monitored, by measuring the flow depth h [m] at each change of the flume slope, with two micrometer screw gauges placed across the patch.

By working out the acquired information, the experiment can be conveniently shown in the Q_s–X–Y space, where Q_s is a dimensional bed load, measured in terms of number of particles passing across the control section over a selected unit of time. The result is shown in Fig. 6. Once the characteristic linear dimension D of the grain mixture is fixed, X and Y are constrained by the relation

$$Y=\frac{\rho v^2}{gS_gD^3}X^2 \tag{7}$$

Accordingly, the straight line on the log X–log Y Shields plane is the set of hydrodynamic conditions explored during the experiment and the curved line on the same plane is the interpretative relation (6) proposed by Pilotti (1995) for the critical adimensional shear stress in shallow viscous flows.

As can be observed, no matter how small the adimensional tractive force Y is during the experiment, there are some particles crossing the control section. This a further demonstration that the concepts of the beginning of the sediment motion and sediment transport must be kept separate. It is interesting to note the rather sharp change of convexity in the pattern of the particles crossing the control section, clearly detectable also in Fig. 7. The sudden increase in the number of particles mobilized as soon as Y is higher than the conventional threshold provided by (6), very likely corresponds to what Yalin and Karahan (1979) define as *carpet motion*. The pattern tends to saturate for the highest Y values due to the limited extent of the experimental patch, whose dimension have been targeted to the identification of the threshold condition and not to the measurement of the bed load.

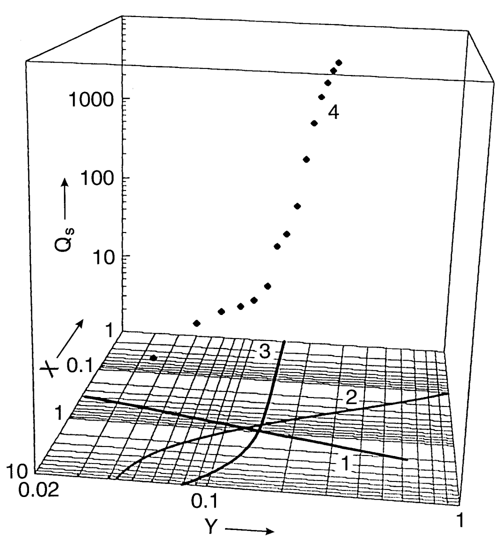

Fig. 6. The Q_s–X–Y space that can be properly used for the identification of the beginning of sediment transport. The dots 4 are the number of particles crossing the control section over the unit of time, in correspondence of the straight line *1*, representing the set of X–Y values explored during the described experiment, analytically given by relation (7). The curve *2* is the classic criterion (5) named after Shields (1936) and the curve *3* is the pattern of the experimental data interpreted by relation (6) for laminar flows. From this figure it is evident that: a) also in laminar condition the concept of inception of sediment transport is conventional; b) for $X<1$ the curve *2* is not confirmed by experimental evidence; c) for $X>1$, there are two separate curves to describe the inception of sediment transport (i.e., *2* and *3*), depending on the regime of the flow

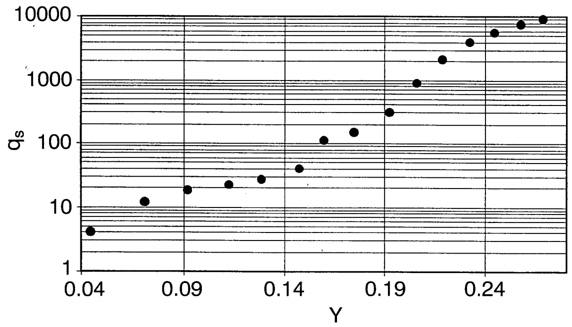

Fig. 7. The q_s–Y relationship observed during the experiment

When $Y=Y(X)$ is lower than (6), several particles still move on the patch. This is not surprising. Even under the assumption of uniform distribution of the shear stress on the bottom, the equilibrium condition of a single grain is determined by its local angle of repose. This, as shown by Miller Byrne (1966) and Kirchner et al. (1990), is stochastically distributed in the range $20^\circ<\varphi<90^\circ$, depending on the relative size of the grains, their departure from sphericity and their angularity.

Accordingly, during the experiment, particles which are placed in an unstable position with respect to the shear stress exerted by the flow, tend to move in order to find a more stable location. This relocation process tends to decrease in time (see Fig. 7, for Y approaching the value 0.15) and comes to an end when the shearing action overcomes the stabilizing force acting on the erodible layer of the bed. Then sediment transport takes place.

5
Conclusions

A solid state linear CCD camera provides an efficient solution to the problem of controlling the motion of grains across a control section, a problem of considerable interest in the field of sediment transport. With respect to a conventional camera, it minimizes the amount of retrieved information, preventing overlapping between the frames. This makes possible to monitor the time evolution of a process, like the movement of sediments on the bottom of a laboratory flume, without any constraint on the amount of acquired data. The proposed approach has been applied to the study of the beginning of sediment transport for $X<20$, providing some interesting information on the mechanics of this phenomenon. Even in the investigated hydrodynamic conditions, the quantitative results show that no definite threshold exists for grains motion.

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