Influence of Earth rotation on the transport in narrow lakes

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RISERVA NATURALE
PARCO LACUALE
MUSEO ARCHEOLOGICO

Is Earth’s rotation relevant in all water bodies?

Does it depend on…

SIZE

WIDTH

→ Rossby radius

$$Ro = \frac{U}{fB}$$

HEIGHT

→ Ekman depth

$$D_E = \sqrt{\frac{2\nu_z}{H}}$$

TURBULENCE

CLOSURE

→constant/space ad time varying

MIXING REGIME

→ Vertical eddy viscosity

→ Horizontal eddy viscosity

→ Density gradients

TIME SCALES

TYPE OF PROCESS

→ Period of external forcings (i.e. wind, inflows, tides)

TRANSIENCE

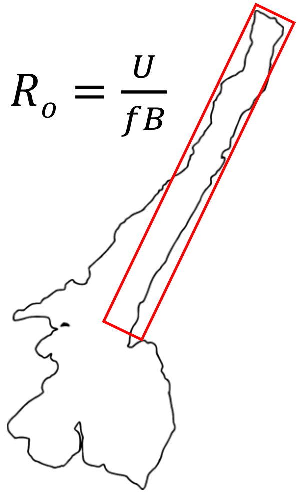
→Spin up time (in numerical simulations)

It is commonly assumed that Earth’s rotation can be neglected in small enclosed basins

Is Earth’s rotation relevant in all lakes?

- What does small mean?

$$R_o = \frac{U}{fB}$$



“Rotational effects need to be taken into account when analyzing the circulation of lakes with horizontal dimensions larger than the ***Rossby radius*** *of deformation R_o (…)”*

Rueda F.J., and J. Vidal (2010) *Currents in the Upper Mixed Layer and in Unstratified Water Bodies*. In: Biogeochemistry of Inland Waters

but also…

“It is not so much the horizontal extent of a basin, but its depth which dictates whether the Earth’s rotation affects the circulation of a water body”

K. Hutter (2011) *The Role of the Earth’s Rotation: Fundamentals* In: Physics of Lakes: V.2 Lakes as Ocillators

- …and does it work for all processes?

some sparks from recent works in narrow alpine lakes:

WIND INDUCED TRANSPORT

Piccolroaz et al. 2018, Importance of planetary rotation for ventilation processes in deep elongated lakes: Evidence from Lake Garda (Italy).

Amadori et al. 2018, Wind variability and Earth's rotation as drivers of transport in a deep, elongated subalpine lake: the case of Lake Garda.

RIVER PLUMES

Pilotti et al. 2018, Evidence from field measurements and satellite imaging of impact Earth rotation on Lake Iseo chemistry.

Earth’s rotation and fluid motion

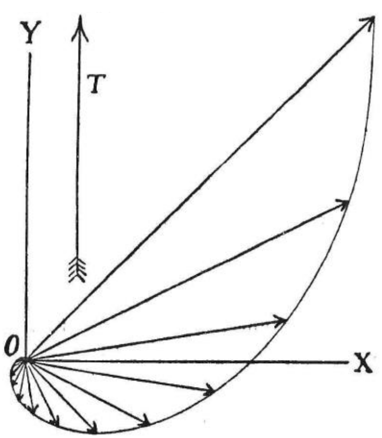
Where it all began…

V.W. Ekman PhD Thesis

On the influence of the Earth’s rotation on ocean-currents.
[Uppsala, 1905]

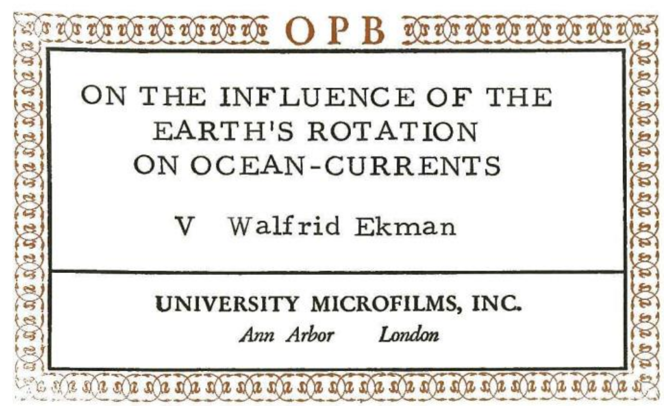
1. Wind stress and Coriolis force
2. Ekman transport
3. Ekman layer → Ekman number
4. Ekman spiral

V.W. Ekman, 1874–1954



Many simplicative assumptions

- *Infinite depth*
- *No pressure gradients*
- *Steady state*
- *Constant turbulence*
- *Constant density*
- *No lateral boundaries*



O P B

ON THE INFLUENCE OF THE EARTH'S ROTATION ON OCEAN-CURRENTS

V Walfrid Ekman

UNIVERSITY MICROFILMS, INC.
Ann Arbor London

Equations of fluid motion

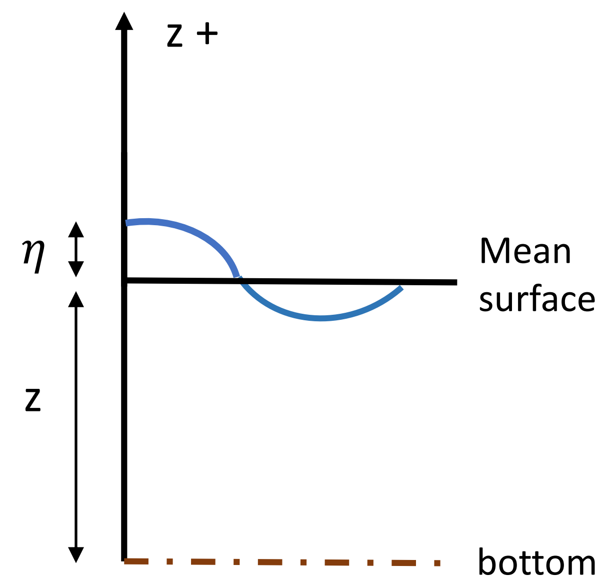
simplified RANS

Momentum

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = fv - \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu_z \dfrac{\partial^2 u}{\partial z^2} + \nu_h \left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) \\ \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} = -fu - \dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + \nu_z \dfrac{\partial^2 v}{\partial z^2} + \nu_h \left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) \\ p = p_0 + \rho g(\eta - z) \end{cases}$$

Assumptions

1. Vertical velocity: $w \ll u, v$
2. Anisotropic turbulence: $\nu_z \neq \nu_h$
3. Hydrostatic pressure
4. Coriolis acceleration: $\begin{cases} (2\underline{\Omega} \times \underline{u})_x \cong -fv \\ (2\underline{\Omega} \times \underline{u})_y \cong fu \end{cases}$
5. Baroclinic effects neglected



Classical analytical solution - Ekman theory, 1905

Assumptions:

1. Steady state: $\frac{\partial}{\partial t} = 0$
2. Non linear advection neglected: $u_i \frac{\partial u_j}{\partial x_i} = 0;$
3. Horizontal turbulence neglected: $\nu_h \frac{\partial^2 u_j}{\partial^2 x_i} = 0;$
4. Constant vertical eddy viscosity ν_z ;
5. Infinite depth z
6. Pressure gradients neglected
7. Wind along y axis

$$\begin{cases} 0 = \quad fv + \nu_z \dfrac{\partial^2 u}{\partial z^2} \\ 0 = -fu + \nu_z \dfrac{\partial^2 v}{\partial z^2} \\ \quad \dfrac{\partial p}{\partial x} = \dfrac{\partial p}{\partial y} = 0 \end{cases}$$

Boundary conditions

Surface (z = 0): wind shear stress

$$\begin{cases} \rho \nu_z \left.\dfrac{\partial u}{\partial z}\right|_{z=0} = \tau_{wind,x} \\ \rho \nu_z \left.\dfrac{\partial v}{\partial z}\right|_{z=0} = \tau_{wind,y} \end{cases}$$

Bottom (z = H/→ ∞):
no slip == vanishing velocity

$$u\Big|_{z\to\infty} = v\Big|_{z\to\infty} = 0$$

Lateral boundaries: commonly neglected assuming the solution to be **far enough from boundaries**

UNKNOWNS:

$$\begin{cases} u(z) \\ v(z) \end{cases}$$

PARAMETERS:

$$\begin{cases} \rho \\ \nu_z \\ \tau_{wind} \end{cases}$$

Classical analytical solution - Ekman theory, 1905

SOLUTION

$$u = V_0 \exp\left(\frac{z}{D_E}\right)\cos\left(\frac{\pi}{4}+\frac{z}{D_E}\right)$$

$$v = V_0 \exp\left(\frac{z}{D_E}\right)\sin\left(\frac{\pi}{4}+\frac{z}{D_E}\right)$$

$$D_E = \sqrt{\frac{2\nu_z}{f}}$$ *Ekman depth*

$$V_0 = \frac{\tau}{\rho\sqrt{\nu_z f}}$$ *scale of velocity*

EKMAN TRANSPORT

Infinite depth $$q_x = \frac{\tau_{wind}}{\rho f}$$

Successive solutions overcoming the simplified original assumptions:

- *Finite depth*
- *Geostrophic balance*
- *Time dependency*
- *Non-uniform turbulence*
- *Density stratification*
- *2D – 3D domains*

Does Ekman-like approach work well in all cases?

Lateral boundaries can be important in narrow – elongated lakes

Toffolon, M. (2013), Ekman circulation and downwelling in narrow lakes.

Ekman type transport in a simplified box domain

A new analytical approach for the 1D problem

Differential problem for complex velocity

$$\nu_z \frac{\partial^2 W}{\partial^2 z} - ifW = gS$$

Vertical boundaries

$$\left.\frac{\partial W}{\partial z}\right|_{z=0} = \frac{T}{\rho \nu_z}$$

$$\left.\frac{\partial W}{\partial z}\right|_{z=-H} = 0$$

Integral conditions for lateral boundaries

$$\boxed{\int_0^H W(z)dz = 0}$$

Closed longitudinal and lateral circulation

UNKNOWNS:

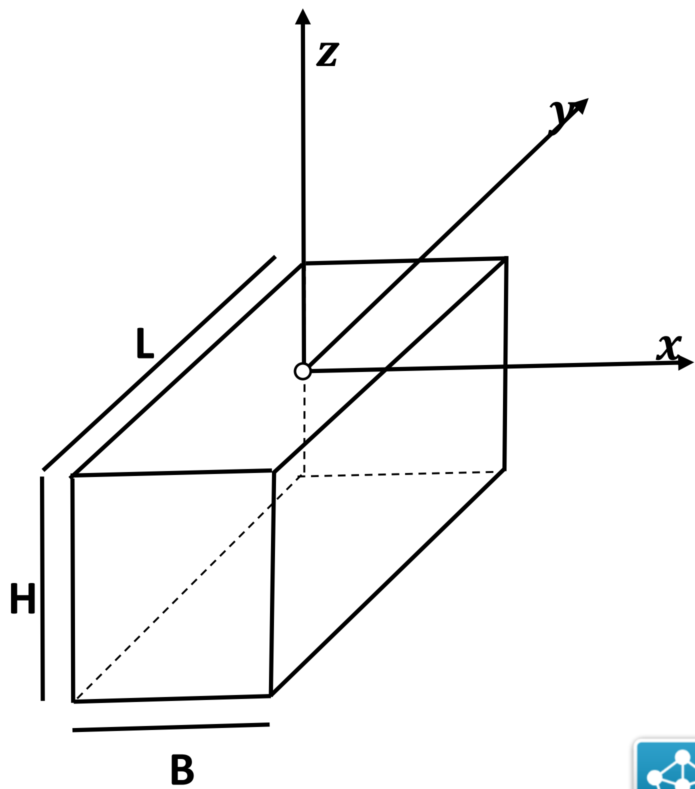
$$\begin{cases} W(z) = u(z) + iv(z) \\ S = \dfrac{\partial \eta}{\partial x} + i\dfrac{\partial \eta}{\partial y} \end{cases}$$

PARAMETERS:

$$\begin{cases} T = \tau_x + i\tau_x \\ \nu_z \end{cases}$$

Ekman type transport in a simplified box domain

Numerical simulations



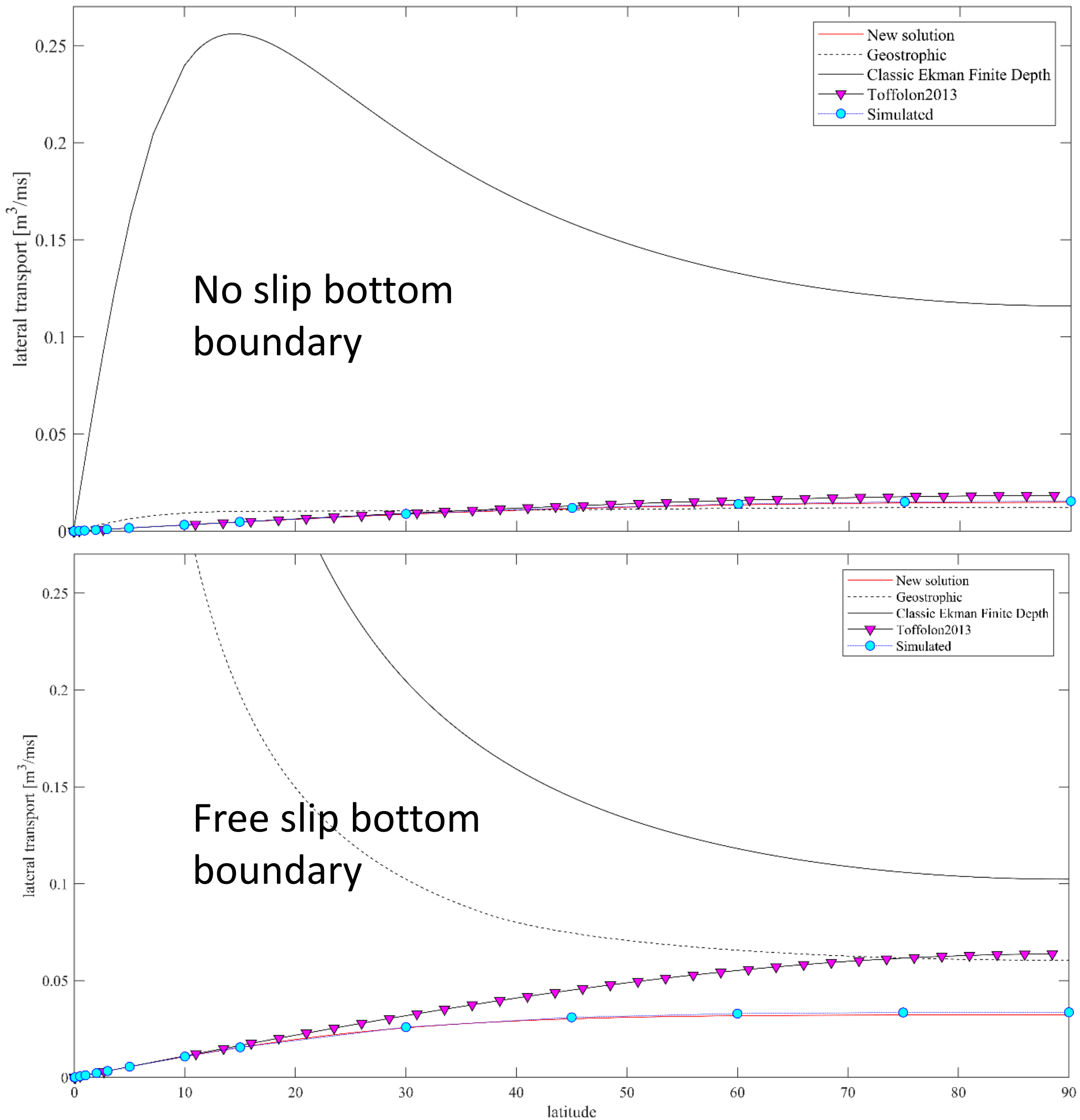
ADDITIONAL SIMULATIONS
Depth effect
H1 = 10 m
H2 = 30 m
H3 = 100 m
H4 = 300 m
H5 = 500 m
Width effect
B1 = 500 m
- $\nu_h = 1\ m^2/s$
- $\nu_h = 10\ m^2/s$
- ν_h =HLES
B2 = 1 km
B3 = 2.5 km
B4 =10 km
B5 = 25 km
vertical turbulence effect
T1 = $5 \times 10^{-3}\ m^2/s$
T2 =$1 \times 10^{-2}\ m^2/s$
T3 = $5 \times 10^{-2}\ m^2/s$
T4 = k-epsilon

All simulations performed with varying latidudes from 0°N to 90°N

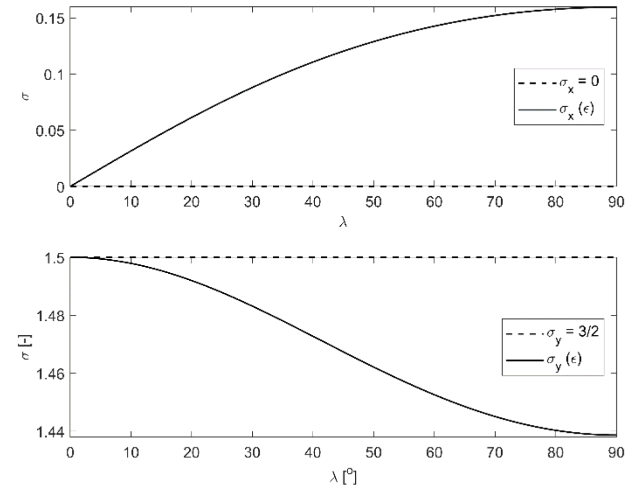
Reference simulations
H = 50 m
B = 5 km
L = 50 km
$\nu_z = 3.73 \times 10^{-2}\ m^2/s$
$\nu_h = 1\ m^2/s$

Ekman type transport in a simplified box domain

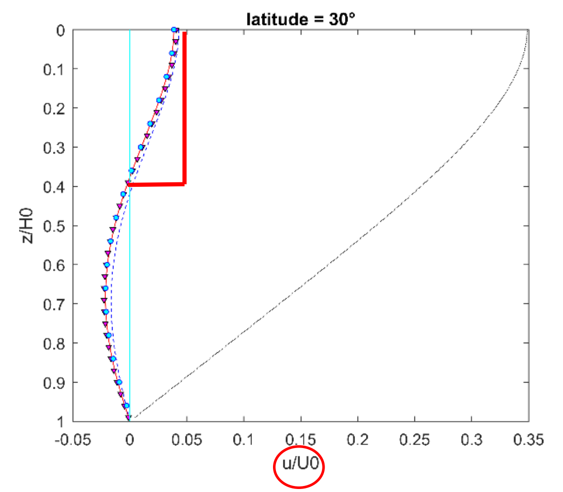
Lateral transport



Surface slope



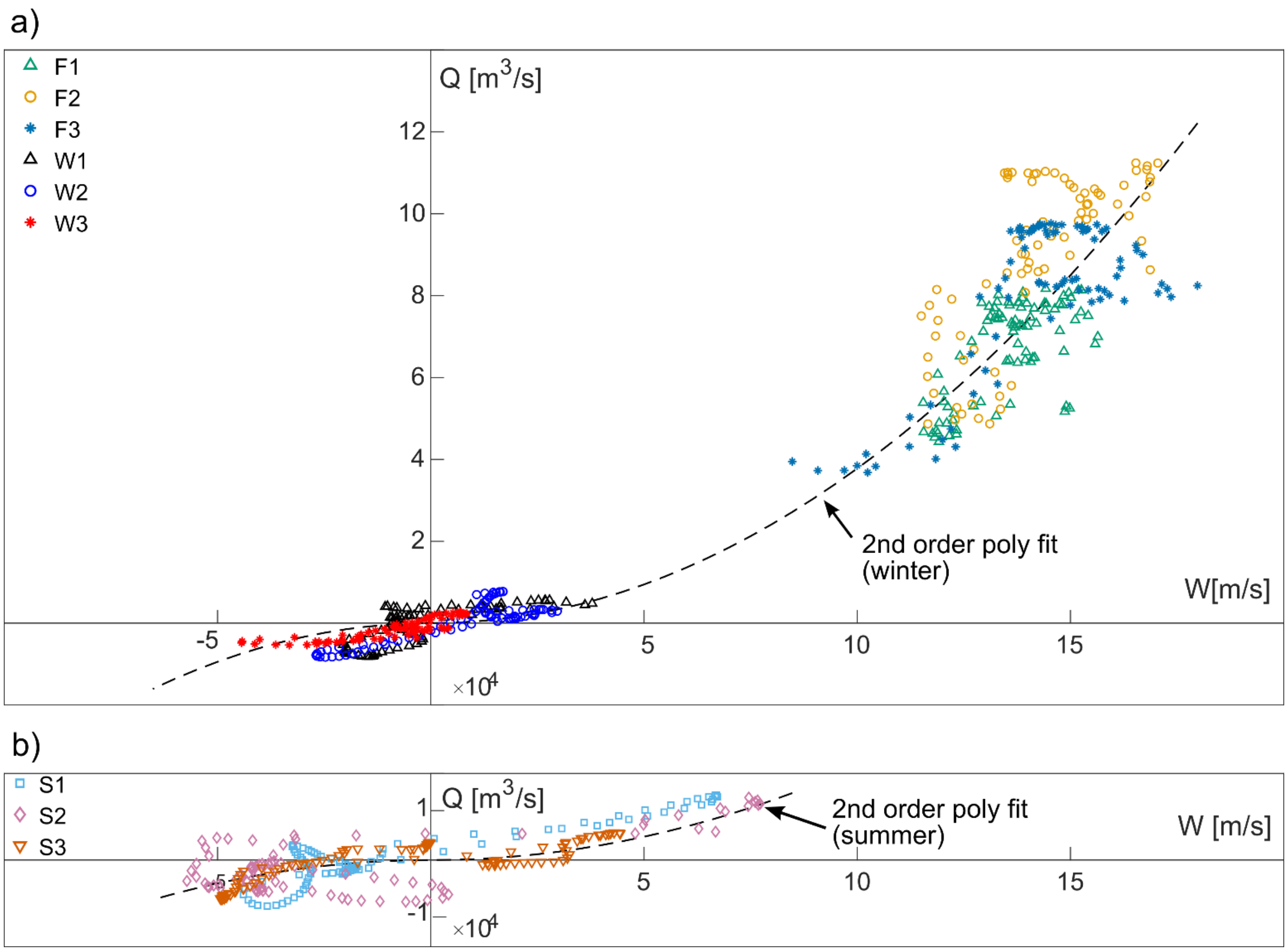
Horizontal velocity profile



From a simplified box domain to real lakes

Lake Garda

Transversal transport vs longitudinal component of wind

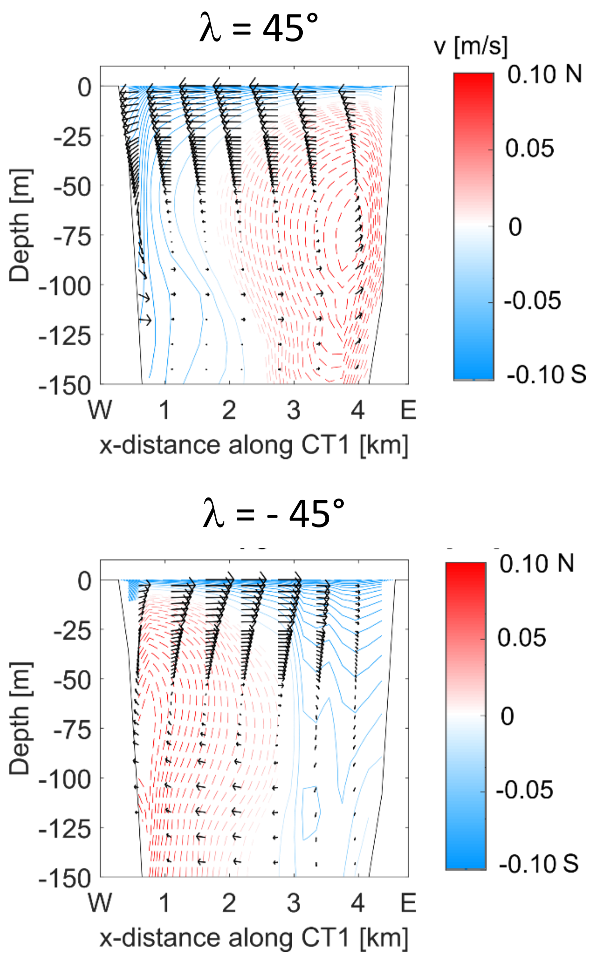


Source: Amadori et al. 2018.

From a simplified box domain to real lakes

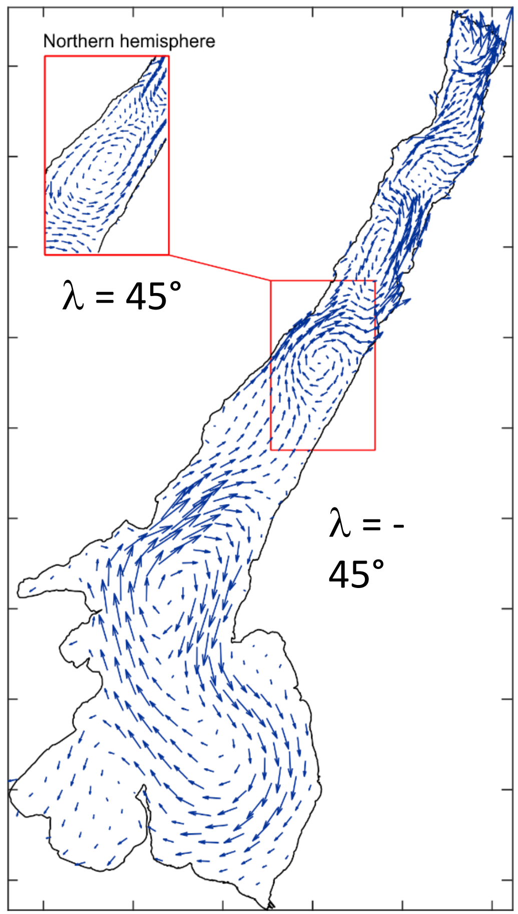
Lake Garda

Winter residual secondary circulations

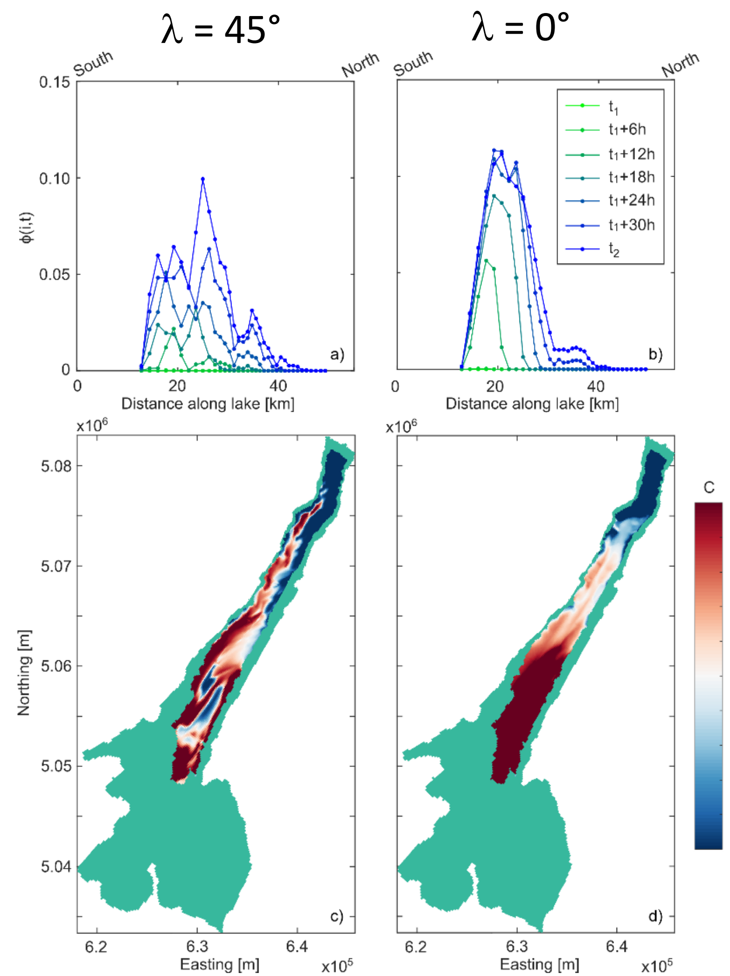


Source: Amadori et al. 2018

Summer residual surface circulations



Deep ventilation processes



Source: Piccolroaz et al. 2018.

Conclusions & Future perspectives

- Theoretical demonstration of the relevence of Earth's rotation also in narrow lakes
- **New analytical approach** suitable for all lakes
- Observational and numerical evidences of Coriolis force effects in real lakes and in **Lake Garda**
- Challenging questions on its effective role in affecting transport and mixing dynamics in real lakes

References

Amadori et al. (2018), Wind variability and Earth's rotation as drivers of transport in a deep, elongated subalpine lake: the case of Lake Garda*, in review for Journal of Limnology.*

Ekman (1905), On the influence of the earth's rotation on ocean currents. *Ark. Math. Astronomi Fys.*, 2 (11), 53.

Piccolroaz et al. (2018), Importance of planetary rotation for ventilation processes in deep elongated lakes: Evidence from Lake Garda (Italy)*, in review for Geophysical Research Letters*

Pilotti et al. (2018), Evidence from field measurements and satellite imaging of impact Earth rotation on Lake Iseo chemistry*, Journal of Great Lakes Research*, 44, 14-25.

Toffolon (2013), Ekman circulation and downwelling in narrow lakes, , *Adv. Water Resour.*, *53*, 76-86.

Thanks for your attention!